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Pricing in spatial classification system in non-symmetric market demand based on the calculations of double interval grey numbers

Modern Supply Chain Research and Applications

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Abstract

Purpose – The contemporary landscape of supply chains necessitates a comprehensive integration of multiple components encompassing production, distribution and customer engagement. The pursuit of supply chain harmony underscores the significance of pricing strategies within the framework of dual-channel distribution, particularly when confronted with the dynamics of asymmetric demand performance.
Design/methodology/approach – This paper delves into a nuanced decision-making challenge anchored in a dual-channel distribution context featuring a retailer and two distinct products. Notably, the retailer's decision-making process employs the computational framework of dual grey numbers, a robust tool for handling uncertainty.
Findings – This study revolves around applying game theory to manufacturers. Each manufacturer presents its aggregated price proposition to the retailer. Subsequently, the retailer identifies the optimal pricing configuration among the manufacturers' aggregate prices while adhering to constraints such as spatial classification and inventory costs. This article's contribution extends to delineating the retailer's capacity to discern the influence of product market potential and the aggregate product cost on the overall demand.
Originality/value – The model's innovation lies in its harmonious fusion of spatial classification, pricing strategies and inventory control. Notably, this novel integration provides a platform for unraveling the intricate interplay between non-symmetric market potential, production costs and cross-sensitivity. The investigation is underscored by the utilization of the double interval grey numbers, a powerful computational approach that accommodates the inherent uncertainty pervasive in the domain. This study fills a gap in the existing literature by offering an integrated framework unifying spatial allocation, pricing decisions and inventory optimization.

Keywords Spatial classification, Pricing, Inventory control, Double interval grey numbers, Non-symmetric demand

Paper type Research paper

1. Introduction

In today's dynamic retail landscape, businesses face multifaceted challenges in optimizing pricing strategies amidst fierce competition, shifting consumer preferences and complex

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supply chain constraints. Retailers must carefully balance pricing decisions with inventory management and shelf space allocation to maximize profitability and customer satisfaction. Inherent uncertainties in demand patterns, production costs, and competitive actions further complicate this intricate interplay between pricing, inventory, and spatial factors.

Consider a supermarket chain grappling with pricing choices for its vast assortment of products. Category managers must determine optimal price points that attract customers and drive sales while accounting for limited shelf space and inventory holding costs. Setting prices too high risks losing customers to rivals, while pricing too low squeezes already thin margins. Effective pricing is not merely a numbers game but a strategic balancing act involving many interconnected variables.

Compounding these challenges, retailers often need complete visibility into consumer demand responses to price changes. Will a 10% discount on a popular item lead to a proportional uptick in sales, or will demand remain relatively inelastic? How will raising prices impact on overall category revenues when substitute products are available? The uncertainty surrounding price elasticity adds another layer of complexity to pricing decisions.

Moreover, retailers must adapt their strategies to asymmetries in market dynamics. Demand for essential goods like bread and milk may be less sensitive to price fluctuations than demand for discretionary items. Retailers need robust frameworks to quantify and manage these differing demand patterns and competitive landscapes across their product mix.

Existing literature has explored various dimensions of pricing optimization, inventory management, and shelf space allocation in retail contexts. However, integrated models that holistically capture the interdependencies between these critical factors while accounting for real-world complexities like demand uncertainty and market asymmetry remain needed.

This study aims to bridge this gap by proposing a novel decision-making framework that unifies spatial classification, pricing optimization, and inventory control. By leveraging grey numbers to encapsulate demand and cost uncertainties and applying game theory principles to model competitive interactions, the approach offers retailers a more comprehensive and realistic tool for navigating pricing challenges in complex market environments.

1.1 Grey number definition

A grey number provides a mathematical representation of the uncertainty, variability, and partial knowledge retailers face regarding consumer demand patterns, production costs from suppliers, competitive pricing changes, and other external or internal factors that influence pricing decisions (Liu and Lin, 2006). For example, a retailer may have a grey demand number ranging from 500–800 units for a product across the potential pricing spectrum, encapsulating their inherent uncertainty based on volatile market conditions. Similarly, suppliers may quote production costs to the retailer as a grey number between $2 and $3.50 per unit, depending on materials, energy, labor, and other variables. The flexibility of the grey number formulation allows the integrated model to capture these demand, cost, and pricing uncertainties within a structured interval range. Grounding the techniques in real-world specifics, the vast array of grey number inputs translates to actionable retail pricing strategy outputs amidst complex supply chain dynamics involving multifaceted tradeoffs between space, inventory, and customer demand factors.

1.2 Elaborating on price sensitivity dynamics

Higher price sensitivity factors for a product indicate that changes in pricing are expected to have a more significant influence on demand levels (Hoch *et al.*, 1995). For example, staple goods often have higher sensitivity parameters around 0.7–0.9 compared to more differentiated specialty products with lower sensitivity in the 0.2–0.5 range (Bolton, 1989). The model incorporates these relative price sensitivities between categories and individual goods in optimizing tradeoffs – acknowledging that the same pricing shift can spur greatly asymmetric demand responses. Beyond product traits, positioning and prominence also impact sensitivity. Items placed in high visibility locations see consumer demand vary more acutely with price changes compared to less emphasized spots (Bezawada *et al.*, 2009). By calibrating sensitivity ratios, retailers can quantify anticipated demand elasticity across their assortments to inform pricing strategies and classification decisions. Computation techniques leverage sensitivity factors to unlock optimization pathways not revealed in symmetrical, static modeling of price-demand relationships.

This study proposes an integrated model incorporating spatial classification, pricing strategies, and inventory control optimization, addressing a gap in the current retail supply chain management literature. The model employs computational techniques of dual grey interval numbers and game theory principles to tackle uncertainty and complexity. The findings provide insights into managing pricing dynamics for asymmetric market demand.

2. Literature review

The efficient allocation of products within storage spaces profoundly impacts individual product sales and the overall sales performance of a retail store. The allocation process directly influences store operating costs, making it a pivotal element of in-store management strategies. Over the past 4 decades, researchers have endeavored to develop nuanced models for categorizing diverse product inventories, yielding various simplified forms (Zhou *et al.*, 2003) that cater to distinct retail environments.

The scientific framework of spatial classification systems pertains to the intricate challenge of optimally distributing scarce spatial resources within a retail establishment. The contours of this challenge morph according to the retail sector, company strategies, vendor relationships, and store layouts (Wang *et al.*, 2015). Striking a balance between product availability and inventory levels is essential. This equilibrium hinges on aligning required products with existing inventory—a cornerstone principle that guides store operations (Urban, 2005).

Simultaneously optimizing spatial allocation entails a delicate interplay between inventory and pricing considerations. In today's landscape, inventory costs greatly influence product positioning, while pricing strategies wield comparable significance within the model. This paper underscores the fusion of spatial allocation, pricing, and inventory optimization within the retail supply chain. To address this multifaceted challenge, this study employs game theory principles and dual interval grey numbers to account for demand uncertainty and inventory control. The resulting model navigates the intricate terrain of spatial allocation, pricing, and inventory decisions within the retail sphere.

This literature review reveals research on spatial classification, inventory management, and pricing decisions in retail supply chains. However, frameworks unifying these areas to optimize retail performance are lacking. The present study proposes an integrated model to address this gap.

Llaguno *et al.* (2022) examined the joint optimization of inventory, pricing, and space allocation in supply chain management contexts, elucidating the interconnections among these factors and their implications for overall performance. (Miranzadeh *et al.*, 2015) delved into an optimization model tailored for inventory management within supply chains involving multiple suppliers and retailers, acknowledging the complexities of diverse supply chain relationships. (Sun *et al.*, 2022) proposed a coordination mechanism for inventory control in decentralized supply chains, addressing the challenges of maintaining efficient operations across distributed entities.

Sajadi and Noori-daryan (2011) presented a mathematical model for optimizing pricing and inventory decisions in production planning, contributing to the intricate balance between pricing and inventory management. (Amelian *et al.*, 2015) explored the joint optimization of pricing and inventory management, specifically in perishable food production, aligning their approach with the perishable nature of goods. (Hatami-Marbini *et al.*, 2020) and (Mahmoudi and Piri, 2013) developed an inventory optimization model for a three-echelon supply chain network, considering the complexities of multilayered supply chain structures.

Emami *et al*. (2014) optimized dynamic pricing and inventory control policies in production systems, tackling the dynamic nature of pricing in manufacturing settings. (Sajadi *et al*., 2016) incorporated pricing and inventory decisions into a production planning model under uncertainty, acknowledging the role of uncertainty in influencing decision-making. (Malekpour *et al*., 2016) jointly optimized inventory levels and production schedules through mixed integer linear programming, offering an integrated production and inventory management approach.

Aiassi *et al*. (2020) integrated pricing, inventory, and operations planning decisions using robust optimization, emphasizing the importance of robust strategies in the face of uncertainty. (Jamshidi *et al*., 2021) applied dynamic pricing and revenue management concepts to quantify entrepreneurial opportunities, highlighting the role of pricing strategies in capitalizing on business potential. (Amelian *et al*., 2022) multi-objective optimization approach for scheduling could be applied to simultaneously optimize pricing, inventory, and shelf space allocation in retail stores.

A comprehensive literature review uncovers a rich tapestry of research related to spatial classification, especially within the retail domain. Notable works include (Hwang *et al*., 2005), who classify storage spatial literature into multi-level branded product scenarios, and (Reyes and Frazier, 2007), who delve into nonlinear numerical programming models for spatial classification allocation with conflicting objectives. Pricing, a pivotal financial dimension, has spurred diverse theories and models over the last 5 decades. (Konur and Geunes, 2016) delve into pricing decisions within a retail chain context, while (Bianchi-Aguiar *et al*., 2018) scrutinize optimal pricing strategies within the food supply chain. Game theory enters the arena through works such as (Jamali and Rasti Barzuki, 2018) exploration of pricing dynamics between green and non-green products and (Tao *et al*., 2019) focus on pricing and inventory policies amid RFID-enhanced inventory management.

The present study lies at the nexus of three pivotal components—spatial classification, inventory control, and pricing decisions—intersecting to drive retail performance optimization. (Martin Herr *et al*., 2006) and (Mahmoodi and Hashemi, 2024) employ game theory to dissect spatial classification and pricing choices within marketing channels, suggesting that unit cost or price adjustments lead to corresponding spatial reallocations. (Wang *et al*., 2015) delve into pricing and spatial classification within a supply chain encompassing a retailer and two manufacturers. Meanwhile, (Reisi *et al*., 2019) propose an approximate solution for retail pricing and shelf allocation to maximize profits without fostering manufacturer competition.

Inventory management enters the discourse as (Singha *et al*., 2017) redefine inventory policies while accommodating spatial classification capacities. (Chang *et al*., 2016) and (Mahmoodi *et al*., 2022a, b) tackles the interplay between desired spatial classification levels, inventory cycles, and ending inventory to enhance annual profits. (Amit *et al*., 2015) offer an optimal spatial classification storage policy by integrating demand-driven external uncertainty into the classroom newsvendor model. (Sony and Suthar, 2018) address inventory challenges tied to unpredictable deterioration, seeking to optimize pricing strategies. (Chen *et al*., 2018) explore traditional pricing and inventory control mechanisms while accounting for time-dependent dynamics.

Recent supply chain management research has continued to explore joint optimization models in pricing, inventory, and spatial allocation decisions. (Mishra and Singh, 2021) developed an integrated pricing and inventory policy for deteriorating items, incorporating promotional efforts and price-sensitive demand. (Rong *et al*., 2020), (Mahmoodi *et al*., 2023 a, b) proposed an optimization approach balancing food quality and supply chain efficiency. (Soni *et al*., 2020) optimized pricing and lot-sizing policies under permissible payment delays. Zhang *et al*. (2019) examined pricing and inventory decisions for perishable goods in retail stores.

Recent literature on pricing models reflects an intensifying focus on tackling uncertainty and integrating supply chain dynamics. Sophisticated approaches have emerged utilizing blockchain (Park *et al.*, 2022), machine learning (Umut *et al.*, 2023), and optimization algorithms (Mahmoodi *et al.*, 2024) and (Hossack-McKey *et al.*, 2023) to navigate increasingly complex retail pricing environments. However, opportunities remain to advance joint pricing-inventory-space frameworks matching the interconnected nature of decisions facing retailers. While Wang *et al.* (2022) and Millar *et al*. (2023, 2024) explore coordinating pricing in the supply chain context, a gap persists in simultaneously addressing inventory and classification constraints. This underscores the value of this study's holistic perspective, accounting for the multiplicity of factors retailers must weigh in optimizing pricing strategies. By consolidating spatial, inventory, and cost considerations, this model delivers an enhanced decision-making toolkit for retailers compared to pricing literature focused solely on isolated decision variables (Mahmoodi *et al*., 2022a, b).

Despite the extensive body of literature exploring various aspects of pricing, inventory management, and spatial classification in retail contexts, there remain notable limitations and opportunities for further research. Existing studies often focus on optimizing isolated components, such as pricing strategies or inventory policies, without fully capturing the complex interdependencies between these factors. Moreover, many models rely on assumptions of symmetric demand patterns and deterministic environments, overlooking the inherent uncertainties and asymmetries prevalent in real-world retail settings. The literature also needs comprehensive frameworks integrating pricing, inventory, and spatial decisions while accounting for the practical constraints and tradeoffs retailers face, such as limited shelf space and inventory holding costs. Furthermore, applying advanced computational techniques, such as grey numbers and game theory, to model uncertainty and competitive dynamics in retail pricing still needs to be explored. These gaps underscore the need for a holistic and robust optimization approach that captures the intricacies of pricing decisions in the face of demand variability, market asymmetries, and supply chain complexities. The present study aims to address these limitations by proposing an integrated model that unifies spatial classification, inventory control, and pricing optimization, leveraging grey number representations and game theory principles to navigate the challenges of retail pricing in complex and uncertain environments. By bridging these gaps, this research contributes to the literature by offering a comprehensive and practical decision-making framework for retailers seeking to optimize their pricing strategies in the face of multifaceted challenges.

These works highlight the complex considerations of managing pricing, inventory levels, product deterioration, promotions, efficiency, and more within modern supply chains. However, there remains an opportunity to advance joint optimization frameworks specifically for retail contexts with demand uncertainty and spatial constraints. The proposed model contributes by uniting computational techniques of dual grey numbers and game theory to balance pricing dynamics, asymmetric demand, spatial classification, inventory control, and multifaceted interconnections impacting retail performance. In this way, it builds upon recent literature on the nexus of these vital retail supply chain management factors.

However, current literature reveals a gap in frameworks unifying spatial allocation, pricing decisions, and inventory control. This study proposes an integrated model to optimize retail performance through these interconnected components.

While existing works have examined joint optimization of pricing, inventory, and spatial factors, this study offers an integrated model designed explicitly for retail contexts dealing with demand uncertainty and inventory constraints. The proposed approach builds on these previous efforts by combining computational techniques of dual grey interval numbers and game theory within a cohesive framework. The model's heart applies game theory for manufacturer pricing while introducing a retailer decision layer to identify optimal configurations adhering to spatial classification and inventory tradeoffs. In this way, the model provides a novel platform based on existing literature that connects pricing dynamics to retail performance factors like asymmetric demand, product market potential, production costs, and their complex interplay.

The present study lies at the nexus of these pivotal components—spatial classification, inventory control, pricing decisions, and their interdependencies—intersecting to optimize retail performance. Yet, the existing literature underscores a discernible gap that this study aspires to bridge. This research combines spatial allocation, pricing, and inventory optimization, unifying these three pillars within the retail landscape. As evidenced by the literature review, attention has been scattered across these themes, often focusing on isolated aspects or twofold combinations. This study addresses this gap by offering a comprehensive framework that unites spatial allocation, pricing, and inventory control to foster a nuanced understanding of their interplay.

2.1 Model distinctiveness and literature contributions

By consolidating the interconnected supply chain considerations of pricing dynamics, inventory control policies, retail spatial layout constraints, and multifaceted tradeoffs, this proposed integrated model provides a significant step forward compared to existing pricing research concentrated on isolated variables. The capabilities unlocked through the novel computational approach combining grey number representations and game theory principles facilitate precise, data-driven decision support for retailers navigating complex markets. Specifically, the model advances literature through:

Inclusion of inventory holding capacity and shelf space limits that restrict pricing freedom based on realistic retail constraints. Handling inherent uncertainty and variability in costs and demand patterns impact pricing. Game theory is applied to mimic competitive manufacturing pricing scenarios. Asymmetric relationships between price shifts and demand responses are quantified.

These differentiating facets underscore the originality of a comprehensive retail-focused decision model that enhances capabilities beyond conventional symmetrical pricing-only considerations. Both researchers and practitioners stand to benefit from the platform to test assumptions, weigh constraints, assess scenarios, and formulate superior dynamic pricing strategy foundations despite market volatility.

The structure of this article is as follows: Section 3 delves into the development of a comprehensive mathematical model. Subsequently, Section 4 undertakes an analytical exploration of the model's underpinnings. Section 5 unfolds a numerical case study, shedding light on real-world implications. Finally, Section 6 concludes this endeavor, encapsulating key insights and avenues for further research.

3. Materials and methods

This section is divided into two parts. Below, in Section 3.1, the calculation of dual grey numbers is introduced twice. Then, under 3.2, the problem model is presented.

3.1 Calculation of dual interval grey numbers

Grey numbers are the central unit of the grey system theory. The meaning of these numbers is related to the propositions. The interval grey numbers can be defined according to the meaning of the propositions (Deng, 1982).

Definition 1. A grey number with the upper and lower boundary of the grey number is called the time interval and is shown as $\theta \in [\theta^-, \theta^+]$. For proposition X, there is information about the Theorem (θ). Due to insufficient information on the theorem or the limited cognitive ability of individuals, people can only obtain a set of probable values of the theorems, and they cannot estimate the exact rate (Liu and Lin, 2006).

(1)θ is an interval grey number in the Theorem $p(\theta)$.

3.1.1 The regret theory based on probable conflict states. In the prerequisite for an independent principle, there are two feelings of regret and joy in preferential decision relationships. Regrets and joy in regret theory are based on the comparison of each state with an ideal point. If the ideal point is considered the reference point, the value of the assessment of the point is less than the ideal point value and regrets in the decision-makers' theory. If the negative point is taken as the reference point, the value of the assessment of the government point is more than the negative ideal point, and decision-makers are happy to be able to evaluate.

According to (Guo *et al.*, 2015), the value of assessing each state S_i proportional to the value of the regret of the ideal point is g_{kh}. The value of happiness relative to the negative ideal is o_{kh} defined as follows:

$$g_{kh}(\theta) = 1 - \exp\left(\delta\left|\rho_{kh}(\theta) - x_h^+\right|\right) \tag{1}$$

$$o_{kh}(\theta) = 1 - \exp\left(\delta\left|\rho_{kh}(\theta) - x_h^-\right|\right) \tag{2}$$

Where $x_h^+ = \max(\underline{x_{kh}}, \overline{x_{kh}}, k = 1, 2, \ldots, m)$ is the positive ideal point, $x_h^- = \min\ (\underline{x_{kh}}, \overline{x_{kh}}, k = 1, 2, \ldots, m)$ is the negative ideal point, and $\delta(\delta > 0)$ is the regret deviation coefficient of the decision-makers. The value of the function of regret-happiness in each state is based on the assessment of decision-makers defined as follows:

$$y_{kh}(\theta) = g_{kh}(\theta) + o_{kh}(\theta) \tag{3}$$

3.1.2 Selection and ordering of the expression of double interval grey numbers in different states. Consider the decision-making view about a proposition's applicable state of conflict. Suppose that the interval grey number $[c_1, d_1]$ and $[a_1, b_1]$ are decision maker's perception of information in the first state of decision; the interval grey number $[c_2, d_2]$ and $[a_2, b_2]$ are decision-makers awareness about the second decision. Because the factors affecting the decision maker are identified in two cases, one is the expected winner, and the other is the decision maker's regret. The interval grey number $[a_i, b_i]$ is related to the second decision. It provides information about revenue or profit and represents the lost data of the decision-maker's behavior, such as psychological and emotional information.

These two decision-making indicators can describe a double interval grey number with the sign of $[c_1, d_1]$, $[a_1, b_1]$, $[c_2, d_2]$, $[a_2, b_2]$. In the current paper, the interval grey number $[a_i, b_i]$ is the confirmation of the decision maker or the degree of support for possible states in different pricing conditions with the spatial classification that will be discussed in the next section. The interval grey number $[c_i, d_i]$ means the degree of regret of the decision maker or the degree of happiness compared to the positive or negative ideal points. For easier comparison, the double interval grey numbers are changed to $([a_1, b_1], [\max\{d_1, d_2\} - d_1, \max\{d_1, d_2\} - c_1])$ and $([a_2, b_2], [\max\{d_1, d_2\}\ - d_2, \max\{d_1, d_2\} - c_2])$.

According to the definition, the double interval grey number is a real number in the interval, the order of the number of possible degrees of grey number of each order is in the range of the interval grey number relative to other interval grey numbers. Then, the possible concept of ordering the interval grey numbers can be confirmed.

Definition 2. If $\otimes_1 = ([a_1, b_1], [c_1, d_1])$, $\otimes_2 = ([a_2, b_2], [c_2, d_2])$ are two interval grey numbers, then the probability $\otimes_1 \geq \otimes_2$ is defined as:

$$\rho(\otimes_1 \geq \otimes_2) = \lambda_1 \max\left\{1 - \max\left(\frac{b_2 - a_1}{b_1 - a_1 + b_2 - a_1}, 0\right), 0\right\}$$
$$+\lambda_2 \max\left\{1 - \max\left(\frac{d_2 - c_1}{d_1 - c_1 + d_2 - c_1}, 0\right), 0\right\}$$
$$+\lambda_3 \max\left\{1 - \max\left(\frac{b_2 + d_2 - (a_1 - c_1)}{b_1 + d_1 - (a_1 - c_1) + b_2 + d_2 - (a_2 - c_2)}, 0\right), 0\right\} \quad (4)$$

where the equations $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$ is confirmed. The relation between $\otimes_1$ و $\otimes_2$ is determined as follows:

(1) if we have $a_i = c_i$ and $b_i = d_i$, $\otimes_1 = \otimes_2$ and $\rho = 0.5$.

(2) If there is an intersection between $\otimes_1$ and $\otimes_2$, $\rho > 0.5$ then $\otimes_1$ is greater than $\otimes_2$ shown as $\otimes_1 > \otimes_2$. When $\rho < 0.5$, $\otimes_1$ would less than $\otimes_2$ shown as $\otimes_1 < \otimes_2$.

3.2 Modeling

Consider a supply chain model with one retailer and two producers with a limited spatial classification. Producers are intended with producers A and B; also, their products are called with the exact identification. Both producers sell their products through retailers. The production cost of products A and B are defined as $C_A(\otimes)$ and $C_B(\otimes)$ respectively. The retailer determines the available spatial classification and delivers orders from producers to fill the spatial classification. The $S(\otimes)$ indicates spatial classification and represents the number of product units stored on the classification. It is assumed that the retailer stores all his inventory in the spatial classification, similar to the demand for that product. $q_A(\otimes)$ is the demand for product A and $q_B(\otimes)$ is the demand for product B with the linear demand function defined as follows:

$$q_A(\otimes) = a(\otimes) - p_A(\otimes) + \theta_A(\otimes)(p_B(\otimes) - p_A(\otimes)) \quad (5)$$
$$q_B(\otimes) = b(\otimes) - p_B(\otimes) + \theta_B(\otimes)(p_A(\otimes) - p_B(\otimes)) \quad (6)$$

where, $p_A(\otimes)$ and $p_B(\otimes)$ are the retail price of products A and B, respectively, $\theta_A(\otimes)$ and $\theta_B(\otimes)$ are the price reciprocal sensitivity parameters for products A and B. Hence, $\theta_A(\otimes), \theta_B(\otimes) \in [0, 1]$ the non-systematic parameters of price reciprocal sensitivity determine when the change in product rate is not the same for products. Parameter a and b are market potential for products A and B. The market potential also reflects consumer sentiment to buy products without any prices.

The linear demand functions are assumed in this model as they provide a straightforward representation of the relationship between price and demand, which is commonly used in the literature (Huang *et al*., 2013) and (Talluri and Van Ryzin, 2006). These functions capture the essential dynamics of price sensitivity and cross-elasticity between products, allowing for tractable analysis and insights. Real-world examples of linear demand functions can be found in various retail contexts, such as consumer packaged goods, where demand for products tends to exhibit a linear response to price changes within a certain range (Tellis, 1988).

This paper applies the three-step decision-making problem. It is assumed that the retailer is the only supply chain supervisor. First, the retailer determines the spatial classification for a product and then provides the information to the producers. Then, the two producers compete to determine the best wholesale price. In general, they simultaneously play games to determine the wholesale price. Finally, concerning the wholesale prices, the retailer selects the retail price. This problem is solved through inverse induction, discussed in the following sub-sections.

3.2.1 Decisions on retail prices. $W_A(\otimes)$ is the wholesale price of product A and $W_B(\otimes)$ is the wholesale price of product B. At this stage, considering the spatial classification and the wholesale price, the retailer decides on the retail price to increase his profit. The net profit of retailers is as follows:

$$\prod_n(\otimes) = (p_A(\otimes) - w_A(\otimes))q_A(\otimes) + (p_B(\otimes) - w_B(\otimes))q_B(\otimes) \tag{7}$$

The decision on the retail price is subject to $q_A(\otimes) + q_B(\otimes) < S(\otimes)$. The retailer encounters the problem of the nonlinear objective function. The optimal retail prices are specified according to Croesh-Kant-Tucker conditions shown in Lemma 1.

Lemma 1. According to the values $S(\otimes)$, $W_A(\otimes)$, $W_B(\otimes)$ the optimal retail price $p_A(\otimes)$ and $p_B(\otimes)$ is as follows:

If $S(\otimes) > S_1(\otimes)$

$$p_A^*(\otimes) = \frac{\begin{array}{l}2a(\otimes)(1+\theta_B(\otimes)) + b(\otimes)(\theta_A(\otimes)+\theta_B(\otimes))\\ +w_A(\otimes)E_1(\otimes) + w_A(\otimes)E_1(\otimes) + w_B(\otimes)E_3(\otimes)\end{array}}{E_5(\otimes)} \tag{8}$$

$$p_B^*(\otimes) = \frac{\begin{array}{l}a(\otimes)(\theta_A(\otimes)+\theta_B(\otimes)) + 2b(\otimes)(1+\theta_A(\otimes))\\ +w_A(\otimes)E_2(\otimes) + w_B(\otimes)E_4(\otimes)\end{array}}{E_5(\otimes)} \tag{9}$$

b) If $S(\otimes) < S_1(\otimes)$

$$p_A^*(\otimes) = \frac{\begin{array}{l}a(\otimes)(3+\theta_A(\otimes)+3\theta_B(\otimes)) + b(\otimes)(1+3\theta_A(\otimes)+\theta_B(\otimes))\\ +S(\otimes)\times F_1(\otimes) + w_A(\otimes)F_3(\otimes) + w_B(\otimes)F_5(\otimes)\end{array}}{4(1+\theta_A(\otimes)+\theta_B(\otimes))} \tag{10}$$

$$p_B^*(\otimes) = \frac{\begin{array}{l}a(\otimes)(1+3\theta_B(\otimes)+\theta_A(\otimes)) + b(\otimes)(3+\theta_B(\otimes)+3\theta_A(\otimes))\\ +S(\otimes)\times F_2(\otimes) + w_A(\otimes)F_4(\otimes) + w_B(\otimes)F_6(\otimes)\end{array}}{4(1+\theta_A(\otimes)+\theta_B(\otimes))} \tag{11}$$

The expression $S_1(\otimes)$ is given in Appendix A, and the auxiliary equations $E_1(\otimes), \ldots, E_5(\otimes)$ and $F_1(\otimes), \ldots, F_6(\otimes)$ are in Appendix B.

3.2.2 Decision on the wholesale price. Each producer identifies his wholesale price. The profit function for producers is defined as follows:

$$\prod_A(\otimes) = (w_A(\otimes) - c_A(\otimes))q_A(\otimes), \tag{12}$$

$$\prod_B(\otimes) = (w_B(\otimes) - c_B(\otimes))q_B(\otimes), \tag{13}$$

Lemma 2. According to the wholesale price $W_A(\otimes)$ and $W_B(\otimes)$, their optimal values are determined:

If $S(\otimes) > S_2(\otimes)$

$$w_A^*(\otimes) = \frac{a(\otimes)(1+\theta_B(\otimes))G_1(\otimes) + b(\otimes)G_3(\otimes) - 8c_A(\otimes)G_5(\otimes) - 2c_B(\otimes)G_7(\otimes)}{G_9(\otimes)} \tag{14}$$

$$w_B^*(\otimes) = \frac{a(\otimes)G_2(\otimes) + b(\otimes)(1+\theta_A(\otimes))G_4(\otimes) + b(\otimes)G_3(\otimes) - 2c_A(\otimes)G_6(\otimes) - 8c_B(\otimes)G_B(\otimes)}{G_9(\otimes)} \tag{15}$$

b) If $S(\otimes) < S_3(\otimes)$

$$w_A^*(\otimes) = \frac{1}{3}\left[2c_A(\otimes) + c_B(\otimes) + \frac{a(\otimes) - b(\otimes) + S(6+\theta_A(\otimes) - \theta_B(\otimes))}{1+\theta_A(\otimes)+\theta_B(\otimes)}\right] \tag{16}$$

$$w_B^*(\otimes) = \frac{1}{3}\left[c_A(\otimes) + 2c_B(\otimes) + \frac{b(\otimes) - a(\otimes) + S(6-\theta_A(\otimes) + \theta_B(\otimes))}{1+\theta_A(\otimes)+\theta_B(\otimes)}\right] \tag{17}$$

c) If $S(\otimes) \in [S_3(\otimes), S_2(\otimes)]$

$$w_A^*(\otimes) = \frac{a(\otimes) + b(\otimes)\theta_A(\otimes) + a(\otimes)\theta_B(\otimes) - S(\theta_A(\otimes)+\theta_B(\otimes)) + c_A(\otimes)(1+\theta_A(\otimes)+\theta_B(\otimes))}{2(1+\theta_A(\otimes)+\theta_B(\otimes))} \tag{18}$$

$$w_B^*(\otimes) = \frac{b(\otimes)(1+\theta_A(\otimes)) + a(\otimes)\theta_B(\otimes) - S(\theta_A(\otimes)+\theta_B(\otimes)) + c_B(\otimes)(1+\theta_A(\otimes)+\theta_B(\otimes))}{2(1+\theta_A(\otimes)+\theta_B(\otimes))} \tag{19}$$

The value $S_3(\otimes) and S_2(\otimes)$ is shown in Appendix A. The auxiliary expressions of $G_1(\otimes), \ldots, G_9(\otimes)$ introduced are shown in Equations (14), (15) are defined in Appendix C.

Based on the optimal wholesale prices available in Lemma 2, the optimal retail price and demand for each product in different cases are as follows:

If $S(\otimes) \succ S_2(\otimes)$, the demand for the two products is as follows:

$$q_A^*(\otimes) = \frac{2(1+\theta_A(\otimes))(a(\otimes)H_1(\otimes) + b(\otimes)H_3(\otimes)) + 2c_A(\otimes)H_5(\otimes) + 4c_B(\otimes)H_7(\otimes)}{H_9(\otimes)} \tag{20}$$

$$q_B^*(\otimes) = \frac{2(1+\theta_B(\otimes))(a(\otimes)H_2(\otimes) + b(\otimes)H_4(\otimes)) + 4c_A(\otimes)H_6(\otimes) + 2c_B(\otimes)H_8(\otimes)}{H_9(\otimes)} \tag{21}$$

Auxiliary terms $H_1(\otimes), \ldots, H_9(\otimes)$ up to H in the above equations are given in Appendix E.

If $S(\otimes) < S_3(\otimes)$, the demand for two products is as follows:

$$q_A^*(\otimes) = \frac{1}{12}\begin{bmatrix} a(\otimes) - b(\otimes) + S(6+\theta_A(\otimes) - \theta_B(\otimes)) \\ -c_A(\otimes)(1+\theta_A(\otimes)+\theta_B(\otimes)) + c_B(\otimes)(1+\theta_A(\otimes)+\theta_B(\otimes)) \end{bmatrix} \tag{22}$$

$$q_A^*(\otimes) = \frac{1}{12}\begin{bmatrix} a(\otimes) - b(\otimes) - c_B(\otimes) + 6S - \theta_A(\otimes)(c_A(\otimes) + S) \\ +\theta_B(\otimes)(S - 2c_B(\otimes)) + c_A(\otimes)(1+\theta_A(\otimes)+\theta_B(\otimes)) \end{bmatrix} \tag{23}$$

If $S_3(\otimes) < S(\otimes) < S_2(\otimes)$, the demand for two products is as follows:

$$q_A^*(\otimes) = \frac{1}{8}\begin{bmatrix} a(\otimes) - b(\otimes) + 2S(2+\theta_A(\otimes) - \theta_B(\otimes)) \\ +(c_B(\otimes) - c_A(\otimes))(1+\theta_A(\otimes)+\theta_B(\otimes)) \end{bmatrix} \tag{24}$$

$$q_B^*(\otimes) = \frac{1}{8}\begin{bmatrix} b(\otimes) - a(\otimes) + 2S(2-\theta_A(\otimes) + \theta_B(\otimes)) \\ +(c_A(\otimes) - c_B(\otimes))(1+\theta_A(\otimes)+\theta_B(\otimes)) \end{bmatrix} \tag{25}$$

These results are utilized to provide Theorem 1, which determines how to specify the optimal spatial classification studied in the next sub-section.

3.2.3 Decision on the size of retail spatial classification. Retailer has $S(\otimes)$ as input data the spatial classification. The retail profit function is calculated follows:

$$\prod_{r}(\otimes) = \prod_{n}(\otimes) - k(\otimes) \times S^2 \tag{26}$$

where, $k(\otimes) \times S^2$, as convex incremental function, indicates the cost of the spatial classification. $k(\otimes)$ is a positive constant that shows the parameter of the spatial classification cost. The spatial classification cost function was first introduced by Kurtuluş and Toktay (2011) [25]. Based on the Lemma 2, the optimal spatial classification is obtained in Theorem 1.

Theorem 1. The optimal spatial classification is determined as follows:

If $K(\otimes) > K_1(\otimes)$

If $K(\otimes) < K_1(\otimes)$

$$S^*(\otimes) = \frac{\begin{bmatrix} a(\otimes)I_1(\otimes) + b(\otimes)I_2(\otimes) - (1 + \theta_A(\otimes) + \theta_B(\otimes)) \\ (c_A(\otimes)I_3(\otimes) + c_B(\otimes)I_4(\otimes)) \end{bmatrix}}{I_5(\otimes)} \tag{27}$$

If $K(\otimes) < K_1(\otimes)$

$$S^*(\otimes) = \frac{\begin{bmatrix} a(\otimes)J_1(\otimes) + b(\otimes)J_2(\otimes) - (1 + \theta_A(\otimes) + \theta_B(\otimes)) \\ (c_A(\otimes)J_3(\otimes) + c_B(\otimes)J_4(\otimes)) \end{bmatrix}}{J_5(\otimes)} \tag{28}$$

$K_1(\otimes)$ is given in Appendix D $I_1(\otimes), \ldots, I_5(\otimes)$ and $J_1(\otimes), \ldots, J_5(\otimes)$ are in Appendix.

By determining the optimal retail space in Theorem 1, the optimal retail price, wholesale price, and demand for channel members are determined as follows. The optimal retail price for products is as follows:

If $K(\otimes) > K_1(\otimes)$

$$p_A^*(\otimes) = \frac{\begin{matrix} a(\otimes)(L_1(\otimes) + U_1(\otimes)) + b(\otimes)(L_3(\otimes) + U_2(\otimes)) \\ +(1 + \theta_A(\otimes) + \theta_B(\otimes))[c_A(\otimes)(L_5(\otimes) + U_3(\otimes)) + c_B(\otimes)(L_7(\otimes) + U_4(\otimes))] \end{matrix}}{U_5(\otimes)} \tag{29}$$

$$p_B^*(\otimes) = \frac{\begin{matrix} a(\otimes)(L_2(\otimes) + U_1(\otimes)) + b(\otimes)(L_4(\otimes) + U_2(\otimes)) + (1 + \theta_A(\otimes) + \theta_B(\otimes)) \\ [c_A(\otimes)(L_6(\otimes) + U_3(\otimes)) + c_B(\otimes)(L_8(\otimes) + U_4(\otimes))] \end{matrix}}{U_5(\otimes)} \tag{30}$$

where the auxiliary terms $U_1(\otimes), \ldots, U_5(\otimes)$ and $L_1(\otimes), \ldots, L_8(\otimes)$ are given in Appendix F.

If $K(\otimes) < K_1(\otimes)$

$$p_A^*(\otimes) = \frac{\begin{matrix} a(\otimes)(N_1(\otimes) + M_1(\otimes)) + b(\otimes)(N_3(\otimes) + M_2(\otimes)) + (1 + \theta_A(\otimes) + \theta_B(\otimes)) \\ [c_A(\otimes)(N_5(\otimes) + M_3(\otimes)) + c_B(\otimes)(N_7(\otimes) + M_4(\otimes))] \end{matrix}}{M_5(\otimes)} \tag{31}$$

$$p_B^*(\otimes) = \frac{\begin{matrix} a(\otimes)(N_2(\otimes) + M_1(\otimes)) + b(\otimes)(N_4(\otimes) + M_2(\otimes)) + (1 + \theta_A(\otimes) + \theta_B(\otimes)) \\ [c_A(\otimes)(N_6(\otimes) + M_3(\otimes)) + c_B(\otimes)(N_8(\otimes) + M_4(\otimes))] \end{matrix}}{M_5(\otimes)} \tag{32}$$

where the auxiliary terms $M_1(\otimes), \ldots, M_5(\otimes)$ and $N_1(\otimes), \ldots, N_8(\otimes)$ are given in Appendix.

According to equations (31) and (32) similar results for wholesale price is obtained as follows:

If $K(\otimes) > K_1(\otimes)$

$$w_A^*(\otimes) = \frac{2c_A(\otimes) + c_B(\otimes)}{3} + \frac{a(\otimes) - b(\otimes)}{3(1 + \theta_A(\otimes) + \theta_B(\otimes))} + (6 + \theta_A(\otimes) - \theta_B(\otimes)) \times O(\otimes) \tag{33}$$

$$w_B^*(\otimes) = \frac{2c_B(\otimes) + c_A(\otimes)}{3} + \frac{a(\otimes) - b(\otimes)}{3(1 + \theta_A(\otimes) + \theta_B(\otimes))} + (6 - \theta_A(\otimes) + \theta_B(\otimes)) \times O(\otimes) \tag{34}$$

where the auxiliary term $O(\otimes)$ is given in Appendix.

If $K(\otimes) < K_1(\otimes)$

$$w_A^*(\otimes) = \frac{c_A(\otimes)}{2} + \frac{a(\otimes) + b(\otimes)\theta_A(\otimes) + a(\otimes)\theta_B(\otimes)}{2(1 + \theta_A(\otimes) + \theta_B(\otimes))} + P(\otimes) \tag{35}$$

$$w_A^*(\otimes) = \frac{c_B(\otimes)}{2} + \frac{a(\otimes) + b(\otimes)\theta_A(\otimes) + a(\otimes)\theta_B(\otimes)}{2(1 + \theta_A(\otimes) + \theta_B(\otimes))} + P(\otimes) \tag{36}$$

where the auxiliary term $P(\otimes)$ is given in Appendix.

The demand for product is as follows:

If $K(\otimes) > K_1(\otimes)$

$$q_A^*(\otimes) = \frac{a(\otimes)Q_1(\otimes) + b(\otimes)Q_3(\otimes) - (1 + \theta_A(\otimes) + \theta_B(\otimes))(c_A(\otimes)Q_5(\otimes) - c_B(\otimes)Q_7(\otimes))}{Q_9(\otimes)} \tag{37}$$

$$q_B^*(\otimes) = \frac{-a(\otimes)Q_2(\otimes) + b(\otimes)Q_4(\otimes) + (1 + \theta_A(\otimes) + \theta_B(\otimes))(c_A(\otimes)Q_6(\otimes) - c_B(\otimes)Q_8(\otimes))}{Q_9(\otimes)} \tag{38}$$

where the auxiliary term $Q_1(\otimes), \ldots, Q_9(\otimes)$ is given in Appendix.

If $K(\otimes) < K_1(\otimes)$

$$q_A^*(\otimes) = \frac{a(\otimes)R_1(\otimes) + b(\otimes)R_3(\otimes) - (1 + \theta_A(\otimes) + \theta_B(\otimes))(c_A(\otimes)R_5(\otimes) - c_B(\otimes)R_7(\otimes))}{R_9(\otimes)} \tag{39}$$

$$q_B^*(\otimes) = \frac{a(\otimes)R_2(\otimes) + b(\otimes)R_4(\otimes) - (1 + \theta_A(\otimes) + \theta_B(\otimes))(c_A(\otimes)R_6(\otimes) - c_B(\otimes)R_8(\otimes))}{R_9(\otimes)} \tag{40}$$

The auxiliary terms are given in the Appendix.

4. An analytic study of parameters

For the expressions of Equations (27) and (28) and the expressions for the optimal demand for the products described in Equations (37) – (40), it is shown that the relation that $q_A^*(\otimes) + q_B^*(\otimes) = S^*(\otimes)$ holds for the optimal spatial classification in Theorem 1.

Because $I_1(\otimes), I_2(\otimes), I_5(\otimes)$ and $J_1(\otimes), J_2(\otimes), J_5(\otimes)$ are positive, the optimal spatial classification will increase with market potential. The rate of increase in demand equals to $I_1(\otimes)/_{I_5(\otimes)}$, when $K(\otimes) > K_1(\otimes)$. $J_1(\otimes)/_{J_5(\otimes)}$ the increase rate of product B is $I_1(\otimes)/I_5(\otimes)$ when $K(\otimes) > K_1(\otimes)$ and $J_2(\otimes)/_{J_5(\otimes)}$ when $K(\otimes) < K_1(\otimes)$.

Since $\partial\left[I_1(\otimes)/_{I_5(\otimes)}\right] \Big/ \partial K(\otimes) < 0$ the increase in the spatial classification cost reduces the positive impact of the market potential in the spatial classification, it also means that if the potential of the product market is constant, retailers can increase total demand by reducing units. Because $I_3(\otimes), I_4(\otimes), I_5(\otimes)$ and $J_3(\otimes), J_4(\otimes), J_5(\otimes)$ are positive, it is found that the desirable spatial classifications decrease with the production cost.

The reduced rate in the demand for product A is $I_3(\otimes)/_{I_5(\otimes)}$ when $K(\otimes) > K_1(\otimes)$ and $J_3(\otimes)/_{J_5(\otimes)}$, when $K(\otimes) < K_1(\otimes)$. If $\partial\left[I_3(\otimes)/_{I_5(\otimes)}\right]\Big/\partial K(\otimes) > 0$, $\partial\left[I_4(\otimes)/_{I_5(\otimes)}\right]\Big/\partial K(\otimes) > 0$, $\partial\left[J_3(\otimes)/_{J_5(\otimes)}\right]\Big/\partial K(\otimes) > 0$ and $\partial\left[J_4(\otimes)/_{J_5(\otimes)}\right]\Big/\partial K(\otimes) > 0$. It is found that spatial classification cost worsens the inverse relationship between production cost and spatial classification. The retailer can increase the total demand by reducing the unit spatial classification cost if the production cost is constant. As a result, it is found that if the retailer cannot affect the potential of the product market and the producer's costs, it can absorb higher customer demand.

Because $K_1(\otimes), K_2(\otimes)$ and $L_1(\otimes), \ldots, L_4(\otimes)$ are positive, the optimal retail price for both products increases with increased market potential. In addition, if $L_1(\otimes) < L_2(\otimes) < L_3(\otimes) < L_4(\otimes)$, it is found that the market potential of the product has a more significant impact on the optimal retail price. With $L_1(\otimes) > L_3(\otimes)$ and $L_2(\otimes) < L_4(\otimes)$, it is found that the impact of product market potential on the retail price of a product is more significant than the impact of the potential of the rival market.

Since $L_5(\otimes) + K_3(\otimes) > 0$ and $L_8(\otimes) + K_4(\otimes) > 0$, if producers can produce at a lower cost, consumers can buy at lower retail prices. There is a positive relationship between the optimal retail price and the market potential and between the optimal retailer and the production cost. Since $\partial\left[L_1(\otimes) + K_1(\otimes)/_{K_5(\otimes)}\right]\Big/_{\partial K(\otimes)} > 0$, $\partial\left[L_5(\otimes) + K_3(\otimes)/_{K_5(\otimes)}\right]\Big/_{\partial K(\otimes) > 0}$ and $\partial\left[L_8(\otimes) + K_4(\otimes)/_{K_5(\otimes)}\right]\Big/_{\partial K(\otimes)} > 0$, the positive relationship between the desirable retail price and the market potential improves, and with the increase in the unit spatial classification cost, the positive impact of the production cost on retail price reduces. Also, retailers can control the impact of market potential or production costs on the retail price by managing spatial classification costs.

Since $Q_1(\otimes), \ldots, Q_4(\otimes)$ are positive, an increase in market potential leads to an increase in the demand for the product and, simultaneously, reduces the demand for the rival. With $Q_1(\otimes) > Q_2(\otimes)$ and $Q_4(\otimes) > Q_3(\otimes)$, when the market potential of a product increases, the total demand rises because the increase in the demand is higher than the reduction in the demand for the rival. Retailers must increase the market potential of a product to have more sales. Since, $Q_5(\otimes), \ldots, Q_8(\otimes)$ are positive, if the production cost increases, demand for that product decreases and demand for the rival increases. An increase in production costs means rise in the retail prices and rise in the retail prices is penalty for consumers.

By considering $Q_5(\otimes) > Q_6(\otimes)$ and $Q_8(\otimes) > Q_7(\otimes)$, Retailers can help the producer to reduce production costs, which increases in demand. Additionally, the low market potential impact of product A (or product cost B) on total demand equals $Q_1(\otimes) - Q_2(\otimes)/_{Q_9(\otimes)}$ (or $Q_4(\otimes) - Q_3(\otimes)/_{Q_9(\otimes)}$). The marginal impact of product cost A (or product cost B) on the total demand equals to $Q_5(\otimes) - Q_6(\otimes)/_{Q_9(\otimes)}$ (or $Q_8(\otimes) - Q_7(\otimes)/_{Q_9(\otimes)}$).With respect to the specific amount of the budget, retailers can use it to improve the overall demand. The two above-mentioned marginal features help the retailer choose one of two strategies: increasing the market potential or reducing the production cost strategy.

5. Numerical example and managerial insights

This section presents a numerical example designed to illuminate the proposed model's multifaceted implications. Its aim is to extract actionable managerial insights by examining the impact of various parameters within the model's framework. First, the impact of market potential on price and significant demand is discussed.

To justify the analyses conducted in this paper, we provide a real-world example demonstrating how retailers can utilize our proposed pricing and spatial classification model. For instance, a supermarket chain can use our model to determine optimal prices for various products, which not only maximizes profit but also optimizes shelf space utilization. As the market potential of a product increases, the demand for that product rises, subsequently requiring more shelf space. Conversely, a reduction in production costs can lead to lower retail prices, which in turn increases product demand. These analyses, compared to real data from reputable retail chains, show that our model can help retailers make optimal decisions by considering spatial constraints and price sensitivity.

The example is investigated for different values of the market potential for products A, B shown as $a/_b$. Accordingly, in Table 1, numerical results are shown for the product's market potential for both the retailer and producers. In Table 2, different values of the production cost ratio of producers A and B for products A, B, have been investigated. Also, Table 3 examines different values of the reciprocal sensitivity ratio of products A and B $\theta_A(\otimes)/_{\theta_B(\otimes)}$. Tables 1–3, $\prod_{r,A}(\otimes)$ and $\prod_{r,B}(\otimes)$ respectively, indicate retail sales of products A and B respectively.

From Table 1, the retail price of each product and the retail spatial classification increase with an increase in the potential market share $a/_b$. A more accurate analysis is shown in Figures 1 and 2. From Figure 1, the retail price of products in the same category increases as the market potential of one of the products in this category grows. Figure 2 shows that an increase in the market potential ratio positively impacts product A production and hurts the demand for product B. The total demand for this category of retailers increases, and the need to hire a more significant classification is felt. In addition, Table 1 shows that a producer with larger market potential has more profit than his rivals. This larger profit can be linked to the dual impact of market potential and wholesale price.

Table 1 shows the impact of the market potential ratio on each channel member's spatial classification, demand, and profit under symmetric sensitivity parameters $\theta_A(\otimes) = \theta_B(\otimes) = 1$. From Figure 3, in non-symmetric demand functions, a company must adopt a mixed strategy, using the market potential and the reciprocal sensitivity of prices to achieve a sure profit. Figure 3 shows that producer A can achieve the same profit level 2 at the points E_1, E_2 or E_3. The point E_1 has a potential low-to-market ratio and reciprocal sensitivity ratio related to E_2 and E_3 with higher potential ratio to market potential and sensitive parameters. This result indicates that a company can adopt different strategies to meet the predetermined profits.

From Table 2, a product with a higher production cost than that of a competitive product provides less profit for the producers and retailers. This mainly relates to lower demand resulting from wholesale prices and retail prices. Figure 4 also indicates that when the ratio of the production cost of product A to that of product B increases, the decrease in the demand for product A is greater than that in demand for product B, and therefore the total demand for retailers decreases, and consequently, less space is needed. From a retail sales viewpoint, the increased production cost less profit. From Table 2, for example, the retailer's profit decreases by 20%, while the production cost ratio A increases from 1 to 2. To maintain profit, producers should reduce their production costs.

The interaction between the cost of production and the sensitivity of the price parameter is discussed. Increasing production costs will reduce the profit for the producer. Figure 5 shows that a producer can use a product differentiation strategy to offset the effect of increasing production costs. As shown in this figure, when the production cost ratio rises from 0.6 to 0.8, the producer's profit decreases by 18%; the sensitivity ratio is 1, but only 16% when the ratio of the parameter to the sensitivity scale is 2. This phenomenon has yet to be reflected in the models presented in previous studies (Kurtulusand and Toktay, 2011), in which the demand function is presented symmetrically.

According to Table 3, we realized that the retailer sets a single retail price for each product when market potential, production costs, and the degree of symmetry of the two products are almost symmetric. The two manufacturers set the same prices, and both products are equally

Table 1. Numerical solution results for different values of $b(\otimes) = 10$, $k(\otimes) = 0.5$, $\theta_A(\otimes) = \theta_B(\otimes) = 1$

	Retailer						Producer A				Producer B			
a/b	$S(\otimes)$	$p_A(\otimes)$	$p_B(\otimes)$	$\prod_{r,A}(\otimes)$	$\prod_{r,B}(\otimes)$	$\prod_r(\otimes)$	$c_A(\otimes)$	$W_A(\otimes)$	$q_A(\otimes)$	$\prod_A(\otimes)$	$c_B(\otimes)$	$W_B(\otimes)$	$q_B(\otimes)$	$\prod_B(\otimes)$
0.2	2.14	8.15	8.08	4.895	4.817	10.65	1.24	2.74	4.18	6.27	2.28	3.74	1.11	1.621
0.4	2.42	8.34	9.65	5.345	6.891	10.94	1.52	2.81	4.51	5.818	2.41	3.81	1.18	1.652
0.6	2.74	8.64	10.35	5.646	7.749	11.35	1.84	3.05	4.82	5.832	2.72	4.05	1.23	1.636
0.8	2.81	9.25	10.65	6.642	8.476	11.65	1.91	3.18	5.13	6.515	3.23	4.18	1.31	1.245
1.0	3.05	9.65	10.94	7.042	8.809	11.88	2.15	3.51	5.45	7.412	3.85	4.51	1.37	0.904
1.2	3.18	10.85	11.35	8.502	9.207	12.11	2.28	4.51	5.66	12.62	4.05	4.82	1.41	1.086
1.4	3.51	11.35	11.65	9.143	9.584	12.35	2.41	4.82	5.76	13.88	4.18	5.13	1.47	1.397
1.6	3.82	12.88	12.31	11.37	10.5	12.88	2.72	5.13	5.94	14.32	4.51	5.45	1.53	1.438
1.8	4.13	14.31	12.97	13.18	10.97	13.31	3.23	5.45	6.18	13.72	4.82	6.32	1.65	2.475
2.0	4.45	14.97	13.35	14.19	11.38	14.35	3.85	6.32	6.25	15.44	5.13	6.77	1.73	2.837

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Table 2. Numerical solution results for different values of $c_A(\otimes)/c_B(\otimes)$, $k(\otimes) = 0.5$, $\theta_A(\otimes) = \theta_B(\otimes) = 1$, $a = b = 1$

	Retailer						Producer A			Producer B		
$c_A(\otimes)/c_B(\otimes)$	$S(\otimes)$	$p_A(\otimes)$	$p_B(\otimes)$	$\prod_{r,A}(\otimes)$	$\prod_{r,B}(\otimes)$	$\prod_r(\otimes)$	$W_A(\otimes)$	$q_A(\otimes)$	$\prod_A(\otimes)$	$W_B(\otimes)$	$q_B(\otimes)$	$\prod_B(\otimes)$
0.2	1.32	4.27	4.41	2.51	1.28	1.93	1.35	0.86	0.99	1.66	0.46	0.28
0.4	1.29	4.30	4.41	2.26	1.36	1.97	1.46	0.89	0.84	1.66	0.50	0.33
0.6	1.26	4.34	4.40	2.02	1.43	1.65	1.57	0.73	0.71	1.70	0.53	0.38
0.8	1.23	4.36	4.40	1.78	1.50	1.52	1.69	0.67	0.59	1.75	0.56	0.43
1.0	1.20	4.40	4.40	1.56	1.56	1.40	1.80	0.60	0.48	1.80	0.60	0.48
1.2	1.17	4.43	4.39	1.38	1.62	1.28	1.91	0.54	0.38	1.85	0.63	0.54
1.4	1.14	4.46	4.39	1.15	1.68	1.17	2.03	0.47	0.29	1.89	0.67	0.60
1.6	1.11	4.50	4.39	0.95	1.73	1.07	2.14	0.41	0.22	1.94	0.70	0.66
1.8	1.08	4.53	4.39	0.77	1.78	1.97	2.25	0.34	0.15	1.95	0.74	0.73
2.0	1.04	4.56	4.39	0.65	1.83	1.88	2.37	0.28	0.10	2.03	0.77	0.80

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Table 3. Numerical solution results for different values of $\theta_A(\otimes)/\theta_B(\otimes)$, $\theta_B(\otimes) = 0.5$, $k(\otimes) = 0.5$, $c_A(\otimes) = c_B(\otimes) = 1$, $a = b = 5$

	Retailer						Producer A			Producer B		
$\theta_A(\otimes)/\theta_B(\otimes)$	$S(\otimes)$	$p_A(\otimes)$	$p_B(\otimes)$	$\prod_{r,A}(\otimes)$	$\prod_{r,B}(\otimes)$	$\prod_r(\otimes)$	$W_A(\otimes)$	$q_A(\otimes)$	$\prod_A(\otimes)$	$W_B(\otimes)$	$q_B(\otimes)$	$\prod_B(\otimes)$
0.2	0.73	4.66	4.63	0.95	1.03	1.46	1.85	0.34	0.29	1.97	0.39	0.38
0.4	0.75	4.64	4.62	1.00	1.06	1.49	1.84	0.36	0.30	1.92	0.39	0.36
0.6	0.77	4.62	4.61	1.04	1.08	1.53	1.82	0.37	0.30	1.88	0.40	0.35
0.8	0.78	4.61	4.61	1.08	1.10	1.56	1.81	0.38	0.31	1.84	0.40	0.33
1.0	0.80	4.60	4.60	1.12	1.12	1.60	1.80	0.40	0.32	1.80	0.40	0.32
1.2	0.82	4.59	4.60	1.16	1.14	1.63	1.79	0.42	0.33	1.76	0.40	0.30
1.4	0.83	4.58	4.59	1.20	1.15	1.66	1.78	0.43	0.34	1.73	0.40	0.29
1.6	0.84	4.57	4.59	1.24	1.16	1.68	1.77	0.44	0.35	1.70	0.40	0.28
1.8	0.86	4.56	4.57	1.28	1.17	1.71	1.74	0.45	0.35	1.67	0.40	0.27
2.0	0.87	4.55	4.57	1.32	1.18	1.74	1.75	0.47	0.36	1.64	0.40	0.25

Source(s): Table created by authors

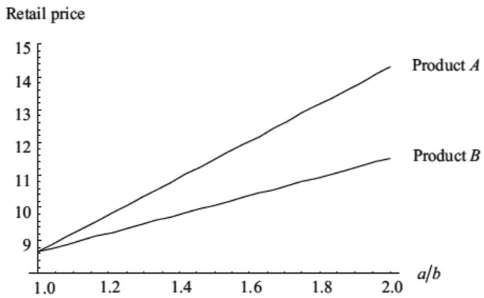

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Figure 1. Retail sales price based on the different product ratios a/b

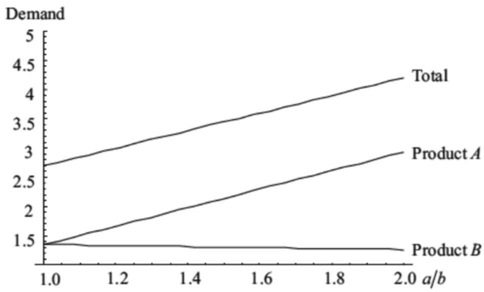

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Figure 2. Demand rate based on the different product ratios a/b

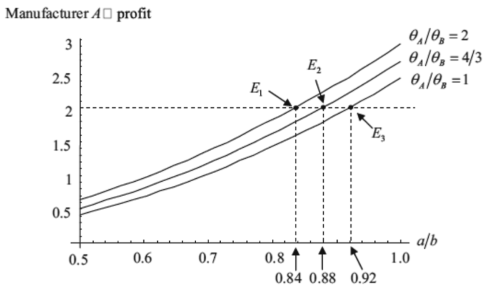

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Figure 3. Profit of Producer A according to ratio a/b, $\theta_A(\otimes)/\theta_B(\otimes)$

divided into retail space classifications. This result is also seen in previous studies (Kurtulus and Toktay, 2011).

Figure 5 illustrates the relationship between the production cost ratio of Product A to Product B and the resulting profit for Producer A. As the production cost of Product A increases relative to Product B, the profit for Producer A decreases, highlighting the sensitivity of profits to changes in production costs. This figure emphasizes the need for producers to manage production costs efficiently to maintain competitive pricing and

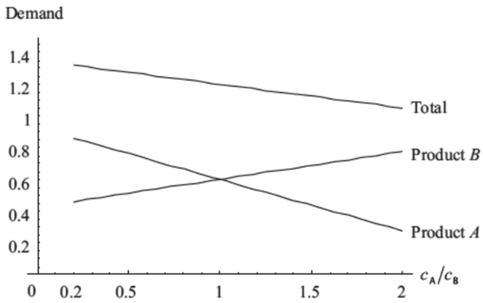

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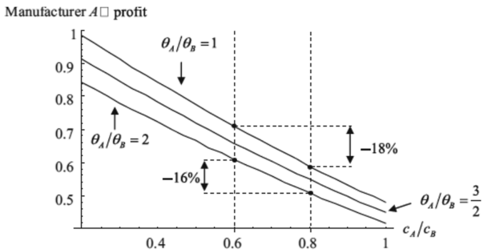
Figure 4. Demand according to different ratios $c_A(\otimes)/c_B(\otimes)$



Source(s): Figure created by authors

Figure 5. Profit of producer A according to ratio $c_A(\otimes)/c_B(\otimes)$, $\theta_A(\otimes)/\theta_B(\otimes)$

profitability. The *y*-axis label "Producer's Profit" has been standardized across all figures for consistency, aligning with the terminology used throughout the manuscript to ensure clarity and coherence in the presentation of results.

5.1 Expanding interpretation of outcomes

The case study quantification offers tangible insights for retail leaders to inform pricing strategy developments and inventory control frameworks applicable to their stores. The presented analyses showcase how adjusting distinct input variables like market demand, production costs, and sensitivity ratios creates cascading impacts on interconnected pricing, classification, and inventory decisions. Notably, the model reveals specific change thresholds in factors like cost fluctuations that could spur retailers to reevaluate pricing given anticipated customer response shifts.

5.2 Emphasizing alignment to pricing dynamics research objectives

The demonstrated pricing alignments connect directly to core modeling research pursuits around managing retail decisions despite uncertainty and constraints. In particular, the computational incorporation of demand variability factors via grey numbers combined with physical shelf space limitations provides a novel approach to capturing the realities retailers face. Quantifying the subsequent interplay between dynamic pricing optimization, inventory availability, and shelf capacities verbalizes the messy intricacies retailers encounter into an actionable decision framework.

5.3 Highlighting practical impacts

Retail leaders can make informed tradeoff decisions by relating case insights around the integrated dependencies between pricing freedom, production costs, inventory holding capacity, and classification space. When facing higher supplier quotes or transport fees squeezing margins, store managers can reference the model operating conditions to weigh pricing shifts accounting for higher sensitivity items and potentially lost sales from display space limits. This mitigates blind price reactions amid volatility.

5.4 Connecting to objectives

In alignment with research goals around balancing pricing decisions amid uncertainty and constraints, the modeled price alignments account for inherent demand fluctuations while weighing the shelf space limits retailers' encounter.

5.5 Implications for practice

When facing margin pressures, management can leverage integrated signals from the model to shape data-informed tradeoff decisions between pricing, costs, turns, and capacities. Reference table impacts help mitigate reactionary guesses.

6. Conclusions

6.1 Key findings

This paper delved into the intricate world of pricing strategies within a spatial classification system, particularly in the context of non-symmetric market demand. The focal point of the study was the decision-making process involving a single retailer and two producers. To tackle the complexity of this problem, the paper introduced the utilization of double interval grey numbers, a mathematical approach combined with game theory principles. The objective was to determine an optimal pricing strategy that carefully balanced various factors, including spatial classification and inventory costs.

The findings of this study provided crucial insights into the complex interplay between product market potential, demand dynamics, and pricing strategies. A significant revelation was a positive correlation between a product's market potential and demand. An increase in the market potential of a specific product had a favorable impact on the demand for that product, but it also had a ripple effect on the demand for other products. This intriguing phenomenon suggested that an upward shift in the demand for one product not only drove the overall demand upwards but also had the potential to stimulate the demand for broader categories of products.

6.2 Managerial implications

The insights from this study have significant implications for businesses navigating pricing decisions within spatial classification systems. Retailers can leverage the model to make data-driven pricing choices in asymmetric market contexts by quantifying interactions between market potential, production costs, shelf space, and price sensitivity.

Furthermore, the study uncovered the intricate relationship between production costs and product demand. As the production costs of a particular item rose, its demand decreased. However, intriguingly, this decreases in demand for one product led to a compensatory increase in demand for competing products. This intricate dynamic demonstrated that the decrease in demand for one specific product was associated with a more pronounced reduction in the need for product classification than the subsequent increase in demand for a competing product.

Additionally, the research illuminated the positive influence of a product's market potential on its retail price. A product with higher market potential could command a higher retail price, emphasizing the importance of understanding and managing market dynamics in pricing decisions.

In the context of retailer decision-making, the study shed light on the retailer's ability to control and manage the impact of market potential and production costs on total demand and retail prices. This management was achieved through carefully considering and controlling spatial classification costs, offering the retailer a strategic lever to optimize pricing strategies.

This integrated pricing model helps retailers manage complex spatial classification and inventory constraints. The novel game theory-based approach models the multifaceted interplay between optimizing pricing factors, physical shelf limitations, and demand uncertainty.

As the numerical analysis demonstrates, managers can leverage the model outputs to inform data-driven pricing decisions amid asymmetric market contexts. Quantifying interactions between market potential, production costs, classification space, and price sensitivity factors enables nuanced strategies to balance profit goals with realistic retail restrictions.

Additionally, the model supports robust pricing strategies by encapsulating the inherent uncertainty in consumer preferences and competitive environments into the grey number inputs. The capacities unlocked by translating variability into defined grey number ranges ensure pricing decisions are insulated from volatility.

6.3 Future research directions

While this integrated model makes several research contributions, ample opportunities remain to build upon the approach within the retail pricing domain. Potential avenues include incorporating behavioral aspects like reference dependence (Mazumdar *et al.*, 2005), expanding competitive scenarios beyond dual manufacturers, applying machine learning to demand forecasting (Zhang *et al.*, 2021), and implementing the model in real-world decision support systems. Testing nonlinear demand responses, stochastic external factors, and congestion effects also provide promising extensions to mimic retail complexity better (Ren *et al.*, 2019). Supply chain researchers can utilize this model's interconnected optimization as a foundation to explore emergent dynamics from enhanced representations of pricing, inventory, and spatial classification factors. Future directions include incorporating behavioral science, broadened scenarios, advanced forecasting, and real-world testing to further build on integrated pricing, inventory, and classification optimization. For academic researchers, opportunities abound to build upon this platform, integrating pricing, inventory, and classification components. Model limitations around linear customer demand functions, equally weighted supplier relationships, and deterministic deterioration rates could be expanded upon by incorporating real-world nonlinearities, channel imbalances, and stochastic elements. Exploring pricing decisions amid alternate competitive and collaborative paradigms may also yield exciting dynamics.

While this integrated model makes several research contributions, ample opportunities remain to build upon the approach within the retail pricing domain. Potential avenues include incorporating behavioral aspects like reference dependence, expanding competitive scenarios beyond dual manufacturers, applying machine learning to demand forecasting, and implementing the model in real-world decision support systems. Testing nonlinear demand responses, stochastic external factors, and congestion effects also provide promising extensions to mimic retail complexity better. Supply chain researchers can utilize this model's interconnected optimization as a foundation to explore emergent dynamics from enhanced representations of pricing, inventory, and spatial classification factors.

Future directions include incorporating behavioral science, broadened scenarios, advanced forecasting, and real-world testing to further build on integrated pricing, inventory, and classification optimization.

In conclusion, the insights garnered from this study are significant for businesses seeking to navigate the intricate pricing landscape within spatial classification systems. By leveraging the relationships between market potential, demand fluctuations, and pricing dynamics, companies can formulate and implement pricing strategies that align with their strategic objectives. This study provides a comprehensive understanding of the nuanced interactions and equips businesses with the knowledge to make informed and effective pricing decisions in a spatial classification context.

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Supplementary material

The supplementary material for this article can be found online.

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