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Research on closed-loop supply chain decision-making of power battery echelon utilization under the scenario of trade-in

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Abstract

Purpose – The management issues of this article, and the author is attempting to address these issues, are as follows: What is the optimal decision of each entity in the closed-loop supply chain for the cascading utilization of power batteries under three government measures: no subsidies, subsidies and rewards and punishments? How do different measures affect the process of cascading the utilization of power batteries? Which measures will help incentivize cascading utilization and battery recycling efforts?

Design/methodology/approach – The paper uses game analysis methods to study the optimal decisions of various stakeholders in the supply chain under the conditions of subsidies, non-subsidies and reward and punishment policies. The impact of various parameters on the returns of game entities is tested through Matlab numerical simulation.

Findings – The analysis discovered that each party in the supply chain will see an increase in earnings if the government boosts trade-in subsidies, which means that the degree of recycling efforts of each entity will also increase; under the condition with subsidies, the recycling efforts and echelon utilization rates of each stakeholder are higher than those under the incentive and punishment measure. In terms of the power battery echelon's closed-loop supply chain incentive, the subsidy policy exceeds the reward and punishment policy.

Originality/value – The article takes the perspective of differential games and considers the dynamic process of exchanging old for new, providing important value for the practice of using old for new behavior in the closed-loop supply chain of power battery cascading utilization.

Keywords Trade-in, Closed-loop supply chain, Power battery, Echelon utilization, Differential game

Paper type Research paper

1. Introduction

Considering the growing severity of environmental issues such as global warming and frequent extreme weather, sustainable development has received extensive attention from the whole society. As an important way to sustainable development, the effective use of resource life cycles has become the focus of attention. The concentrated use of renewable resources and strong industrial connection in the new energy vehicle sector has become a leading field to explore the maximization of the value of the resource life cycles and from the perspective of the whole life cycle, it is not only a large consumer of energy, rare materials and carbon emitters but also an industry with the largest upgrading space for re-usability and environmental protection, so undoubtedly, it has become the vanguard of industrial green development (Ke and Cai, 2018). As its core and the most recyclable component, the power

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battery, the amount of its scrap rate rises continuously, as the new energy vehicles develop rapidly. According to GGII, by 2025, China is expected to possess 137.4 GWh of retired batteries overall, and 960,000 tons of trash batteries will need to be recycled. Most of the power batteries retired from automobiles still have 50% space of their service life to reuse, and directly entering the disassemble step will cause a lot of waste. According to this, the echelon utilization has become an important part of enhancing the value of the industry. However, at present, the standardization system for power battery recycling, echelon utilization, disassembling and re-manufacturing has not yet been formed, and there is no unified standard of recycling methods and echelon utilization for the businesses in the chain ought to submit an application that means the closed-loop supply chain management issue must be resolved immediately in order to achieve effective utilization of power batteries throughout their life cycle.

At present, Japan, the USA and other nations have explored the use of echelon utilization; additionally, they aggressively employ the trade-in technique to pique customers' interest in recycling spent power batteries. However, in terms of trade-in, the impact of this method to achieve effective utilization in the whole process of battery echelon utilization needs to be tested. In addition, in order to encourage efficient recycling, the government subsidizes consumers who use trade in methods, and so forms a trend of echelon utilization and re-manufacturing with subsidies, thereby improving the effective use of resources. The dilemmas faced by the current power battery trade-in mainly include the following five aspects: insufficient awareness of recycling, competition from small workshops, low economic efficiency and imperfect policies and regulations. Among them, the formulation and improvement of policies and regulations can effectively alleviate the problems caused by the first four dilemmas. Therefore, this paper studies the trade-in of power batteries from the perspective of government incentive policies.

Implemented on August 1, 2018, the "Interim Measures for the Management of the Recycling and Utilization of New Energy Vehicle Power Batteries" proposed for the first time in China to use measures such as buyback, trade-in and subsidies to motivate customers to turn in their used batteries. It made trade-in as a recycling method for the public. On August 19, 2021, China formulated the "Management Measures for the Echelon Utilization of New Energy Vehicle Power Batteries," the sense that, from the standpoint of enhancing the overall resource use, it improves the management of echelon utilization of new energy power vehicle firms. In this context, some automobile companies have joined the company of power battery trade-in. Companies led by China Tower and BYD actively responded to the call for power battery trade-in while promoting scale group and large-scale of echelons utilization. In order to investigate the entire value chain of echelon use, new energy vehicle firms like Fengfan, BAIC New Energy and NIO have also progressively included the "battery swap mode" into their new development strategies.

2. Literature review

One of the key topics of current research is how to use trade-in to increase the power batteries' overall life cycle value. Replacing old batteries with new ones is not only an important way of recycling power batteries but also takes into account consumer needs and designs a closed-loop supply chain guided by consumer replacement. Starting from reality, it links the government, enterprises and consumers. In terms of supply chain research in the context of trade in, Huang (2018) analyzed the closed-loop supply chain with trade-in in the context of retail competition. Miao *et al.* (2017) investigated the effects of various players' collecting on the trade-in scenario's closed-loop supply chain. Zhu *et al.* (2016) evaluate the effects of trade-in duopoly competition on closed-loop supply chains. Xiao *et al.* (2017) additionally examined and contrasted retailer-led trade-in as opposed to manufacturer-led trade-in and found that manufacturers or retailers may be more profitable if they do not offer their own trade-in programs but rather a free ride. A few academics take government subsidies into account when examining trade-in supply chains. Regarding government subsidies for those involved in various supply chains, Liu and Guo (2021) discussed the impact of subsidizing different supply chain entities and scale effects on each node of the supply chain. Zhao *et al.* (2016) examined how consumers and remanufacturers share subsidies when making pricing decisions for remanufactured products. Li *et al.* (2018) established a two-stage model of competition between manufacturers and remanufacturers under the scenario of government subsidies and studied consumers' trade-in behavior. Shu *et al.* (2018) thought about government subsidies and a carbon tax in addition to the trade-in. According to studies, government subsidies and suitable carbon taxes can reduce carbon emissions while raising company profits. In the issue of competition for the old for the new, many scholars have also launched a fierce discussion. Cao *et al.* (2018) and Ma *et al.* (2013) studied from the viewpoint of whether enterprises should authorize third-party remanufacturing. Cao *et al.* (2018) argue that authorization is always a better option for manufacturers when there is a significant unit trade-in discount and a high rate of old product recycling; authorization should only be used when the durability of the product is relatively high. Ma *et al.* (2013) argue that it is advantageous for the manufacturer to authorize the remanufacturer to carry out remanufacturing, but from the remanufacturer's point of view, he does not always need to accept the manufacturer's authorization. Zhang *et al.* (2022) created a closed-loop supply chain with "trade-in" and "trade-in" based on consumer preferences for low-carbon consumption and government subsidy policies and contrasted and examined the effects of various subsidy programs and consumer preferences for low-carbon consumption on manufacturers' decisions to reduce emissions, closed-loop supply chain performance and environmental effects. Desai *et al.* (2016) looked at trade-in across categories. In this paper, a cross-category trade-in model is established in the competitive environment of the two companies and the optimal result is obtained. It can be seen that trade-in is widely used as a form of recycling, and many scholars have studied trade-in in combination with government policies such as subsidies.

Few scholars have applied the trade-in to the stepwise utilization of power batteries, while the literature related to power batteries is currently a hot topic of research. In accordance with industrial demands, research on power battery cascade use has also been conducted gradually, mainly focusing on the following aspects. Firstly, cascade utilization of power batteries could reduce secondary pollution. Zeng *et al.* (2014) presented the structure and composition of lithium-ion batteries, outlined the current state of the recycling process for waste lithium batteries and compiled all of the information that was available regarding the pre-treatment, secondary treatment and deep recycling procedures of lithium-ion batteries. The existing recycling process's issues and future opportunities are examined, and it is concluded that battery recycling is necessary, which can not only reduce energy consumption and the shortage of scarce resources but also eliminate harmful and unnecessary pollution, so that the industry with lithium battery consumption can develop in a sustainable direction. Ramoni and Zhang (2013) summarized previous research, they tried to improve the performance of old batteries by removing the electrolyte film on the electrodes using chemical and physical methods and formulated a more reasonable end-of-life strategy (EOL) for the batteries that have been eliminated in electric vehicles in an effort to transform the automotive industry into an environmentally friendly industry. At this stage, China still lacks a flawless and rational method for recycling power batteries, and there is no unified standard for the standardization of the system, so it causes an unequal profit distribution among businesses involved in power battery recycling and makes the unification and standardization of the power battery recycling process more difficult. Kang *et al.* (2013) found that there are a lot of dangerous and poisonous materials in used batteries, which will not only have an impact on the environment; however, if they are not disposed of appropriately, they may also endanger human life and health. According to Ordonez *et al.* (2016), the recycling of precious metals and hazardous substances should be taken into account according to changes in the raw materials and technologies related to battery manufacturing, rather than being fixed or modeled in recycling-related technology. Secondly, the internal power battery cascade usage techniques. Gu *et al.* (2018) developed a three-stage power battery recycling model and discovered that each supply chain participant's revenues are significantly impacted by the cost of recycling retired batteries during the cascade use process. Jiao *et al.* (2022) explored the pricing decisions of various entities in the closed-loop supply chain of power battery cascade utilization and the emission reduction level of vehicle manufacturers under the carbon trading policy. Zhang and Chen (2021) examined how scale effects and government subsidies affected each participant's revenue in the closed-loop power battery supply chain. Zeng *et al.* (2015) studied the obstacles and problems in the recycling process of power batteries in China and finally, found that the lack of laws and regulations, the imperfection of the recycling system and the low level of recycling technology are the biggest challenges in China, and the study proposed that the recycling system can only be improved by extending the responsibility of producers to the recycling system. Webster and Mitra (2007) investigated how recycling is affected by policy. Thirdly, the choice of recycling routes for the power battery closed-loop supply chain. Mu *et al.* (2021) discovered that increasing the level of enterprise cooperation promotes the recycling and reuse of power batteries. Zhu and Yu (2019) analyzed the effect of moral hazard as a foundation for researching supply chain channel selection. Umangi *et al.* (2020) examined how demand, ideal selling prices and receipt rates were affected by self-buyback prices and cross-channel recycling prices in closed-loop supply chain structures run by various vendors. Gong *et al.* (2018) examining how government regulations affect the channels via which used power batteries are recycled. We talked about which recycling channels are most effective based on individual member preferences. Harper *et al.* (2019) sorted out the methods and policy measures of lithium battery recycling today, classified the methods and measures of lithium battery reuse and pointed out that there is still a lack of more effective and scientific recycling processes in China, and the lack of recycling processes has a great impact on the improvement of the environment and the economic effects of recycling, and the study also made predictions about the future recycling trend. Liu and Ma (2021) established a closed-loop supply chain model of power batteries from the perspectives of four modes: no subsidy, subsidized recyclers, subsidized cascade users and subsidized manufacturers, It investigated how recycling scale effects, subsidy objects and subsidy amounts affected each supply chain node as well as the distribution of profits.

Existing literature mainly focuses on power battery recycling strategies and how to subsidize the subjects in the supply chain, while the integration of power battery secondary utilization and trade-in into the closed-loop supply chain has not been studied in depth, but from the perspective of current practice, this integration has gradually become the main mode of operation. The influence of the government on the stepwise utilization of power batteries is multi-faceted, the development of government policies and regulations of the environment, the responsibility of environmental protection, the implementation of sustainable development strategies, the enhancement of consumer awareness of environmental protection and the cooperation of upstream and downstream enterprises in the supply chain require appropriate government intervention. Based on this perspective, this study examines the decision-making of the subjects under three different closed-loop supply chain scenarios, namely, without subsidies, with subsidies and with incentives and penalties, in the context of the integration of trade-in and step-up utilization, with the aim of encouraging the healthy and sustainable development of the power battery market as well as the efficient utilization of retired power batteries and the enhancement of step-up utilization.

3. Game model

The closed-loop supply chain of power battery stepwise utilization mainly considers a single subject within the supply chain, i.e. the manufacturer, distributor and stepwise utilizer as a single subject and the consumer consists of electric vehicle consumers (power battery consumers) and stepwise utilization consumers. There are many ways to recycle power batteries, but the most effective method is the secondary utilization. Manufacturers produce new power batteries and sell them to dealers at wholesale prices and, at the same time, recycle used power batteries that cannot be utilized for laddering for remanufacturing. The distributor sells new power batteries to the consumer market and takes on the responsibility of recycling used power batteries for trade-in, sells power batteries with 30%–80% of remaining power to the ladder utilizer and sells used power batteries that cannot be ladder utilized to the manufacturer for remanufacturing. The recycled power batteries will be disassembled and reorganized by the ladder utilizer to become ladder-utilized products and put into the ladder-utilization market for reuse. In addition, the ladder utilizer will decide its own level of effort according to the level of market recycling in the supply chain and the level of recycling effort of each subject and recycle the used power batteries sold and used by the recycler. Finally, all the used power batteries in the supply chain are returned to the manufacturer for remanufacturing. Therefore, this paper adopts the method of differential game to analyze the decision-making process of the closed-loop supply chain of power batteries in terms of the gradient utilization. Its operation process is shown in Figure 1.

3.1 Variable description and model assumption

The components of the dual closed-loop supply chain that functions in the trade-in scenario described above make decisions in the following order: first, the manufacturer decides the price f of recycling waste power batteries and the degree of recycling efforts made by manufacturers x_m; furthermore, the distributor determines the level of trade in rebates given to consumers p and their efforts in recycling the old for new products x_s and lastly, the cost of the tier utilization products offered to tier utilization consumers is decided by the tier utilization provider p_t and the level of recycling efforts made by the tier utilization provider x_t. The density of batteries and the products processed are different between the gradient utilization and remanufacturing. The gradient utilization is to sort and reorganize waste power batteries with a battery density between 30 and 80% into gradient utilization products to be applied to areas with lower electric energy, while the remanufacturing is to dismantle

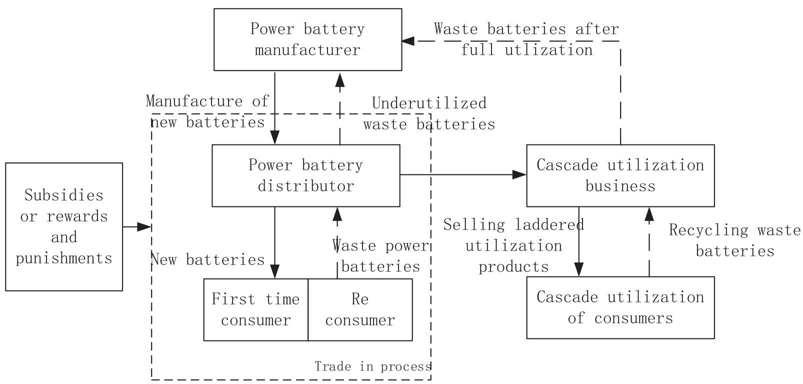

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Figure 1.
Double closed loop supply chain model for power batteries considering cascade utilization

and recycle batteries with a performance of less than 30% and then, extract the metal resources therein to process new power batteries. The waste power batteries recovered by the manufacturer from distributors are low-energy density waste power batteries that cannot be utilized in a cascade manner. The waste power batteries recovered by the manufacturer from a cascade user are waste power batteries that have already passed the cascade utilization stage and cannot be utilized in a cascade manner. The waste power batteries recovered from two sources are essentially power batteries that cannot be reused in a cascade, and the main purpose of recycling is to extract useful metal resources for regeneration. Therefore, the price of discarded power batteries recycled by manufacturers from both sources is the same f. Table 1 displays the closed-loop supply chain's symbolic meaning.

In Table 1, the echelon utilization rate is the proportion of used power batteries recycled by dealers that can be subjected to gradient utilization products to all used power batteries, which is replaced by the demand in the gradient utilization market D divided by the total recycled volume q_{tn} when constructing the model.

In this section, this study aims to simplify a few intricate situations without sacrificing the core of the issue. Initially, the following presumptions must be made:

Assumption 1. In addition to being risk neutral, the manufacturer, recycler and echelon usage merchant are all subject to full information conditions.

Variable	Description	Variable	Description
ω	Unit wholesale price of new power batteries	c_m	Unit manufacturing cost of new power batteries
p_1	Unit retail price of new power batteries	c_r	Unit remanufacturing cost of waste power batteries
$p(t)$	Rebates offered by dealers to consumers for trading-in	c_1	The unit recycling cost of echelon utilization merchants
$n(t)$	Cascade utilization effort level	N_0	Potential market size of initial consumers
p_2	The retail price per unit of discarded power batteries sold by distributors to secondary users	N_1	Potential market size of trade in consumers
p_t	The retail price per unit of high-energy density batteries sold by tier manufacturers	δ	Consumers evaluate discounts on old products
f	The price of manufacturers recycling used batteries	θ	Consumer willingness to pay for new products
o	Difficulty coefficient of echelon utilization	$x_m(t)$	The level of recycling efforts at time t made by the manufacturer
A	Difficulty coefficient of recycling and re-manufacturing	$x_s(t)$	The level of recycling efforts at time t made by the dealer
B	Difficulty coefficient of trading-in	$x_t(t)$	The level of recycling efforts at time t made by the echelon recycling dealer
g	The unit reward (or punishment) amount under reward and punishment measures that is higher (or lower) than the government's preset utilization rate of tiers	φ	The echelon utilization rate of retired power batteries $\varphi = D/q_{tn}, \varphi \le 1$
φ'	The echelon utilization rate set by the government under reward and punishment measures	q_{tn}	Trade-in consumer demand
D	The echelon utilization consumer demand		

Source(s): Table created by authors

Table 1. Model symbols and meanings

Assumption 2. Referring to reference (Han *et al.*, 2017), customers' desire function for new items in the market for electric vehicles is: $q_n = N_0 \int_{p_1}^{1} 1d\theta = N_0(1-p_1)$ and consumers' demand function for supplies trading is: $q_{tn} = N_1 \int_{\frac{p_1-p}{1-\delta}}^{1} 1d\theta = N_1\left(1-\frac{p_1-p}{1-\delta}\right)$.

Assumption 3. Under reward and punishment measures, if the ratio of battery recycling and sales to secondary users by dealers (echelon utilization rate) exceeds (is less than) the prescribed amount, the government will give a reward (punishment) for each unit of battery. The expression for the reward (punishment) that dealers receive from the government is. $g(\varphi-\varphi')q_{tn}$.

Assumption 4. Assuming that the recycling costs of manufacturers and distributors are a quadratic function of the degree of recycling efforts of each entity, and the cascading utilization costs of tier utilization merchants are a quadratic function of their difficulty in cascading utilization. The recycling costs of manufacturers and distributors as well as the cascading utilization costs of secondary users, can be expressed as (Tian and Liu, 2011):

$$C(x_m(t)) = \frac{A}{2}[x_m(t)]^2;$$

$$C(x_s(t)) = \frac{B}{2}[x_s(t)]^2;$$

$$C(x_t(t)) = \frac{o}{2}[x_t(t)]^2.$$

Among them, $C(x_m(t))$、 $C(x_s(t))$、 $C(x_t(t))$ are the recycling costs of the manufacturer and distributor at the moment, as well as the cascading utilization costs of cascading users. The difficulty coefficient of recycling and remanufacturing for manufacturers, the difficulty coefficient of exchanging old for new recycling for distributors and the difficulty coefficient of cascading utilization for secondary users.

Assumption 5. Power batteries' echelon usage level is a dynamic process (Tian and Liu, 2011).

$$\dot{n}(t) = \gamma x_m(t) + \eta x_s(t) + \alpha x_t(t) - \beta n(t)$$

Among them, γ and η, respectively, represent the coefficients of the impact of the recycling efforts of manufacturers and distributors on the level of echelon utilization; α denotes the coefficient of influence between the hierarchical usage level and the merchant's level of hierarchical utilization and β indicates the natural decline rate.

Assumption 6. In the cascading utilization market, the waste power battery demand function is: $D(p_t(t), n(t)) = (k - lp_t(t))n(t), p_t < p_1$ (Liu *et al.*, 2023). The market for echelon usage is distinct from the market for universal reuse. The market for cascading utilization's need for high-energy density waste batteries has no bearing on the power battery industry's need for new batteries. Manufacturers can also recycle all of the discarded batteries with high energy density that they sell for a unit cost of.

Assumption 7. Assuming that manufacturers, distributors and tier players all have the same percentage off at all times $\rho(\rho > 0)$ and that suppliers, retailers, manufacturers and other stakeholders all want to reduce emissions as much as possible while maximizing their own profitability.

Assumption 8. The demand for high-energy-density waste batteries is not significant in the market of the developing industry of the step-up utilization. The demand for high-energy-density batteries $D(p_t, n)$ in the step-up industry is met entirely by the amount of batteries recycled in the electric vehicle market. High-energy-density batteries that cannot be reused in the used batteries collected by dealers are returned directly to manufacturers for remanufacturing. Therefore, the number of used power batteries sold by dealers to manufacturers is $q_{tn} - D(p_t, n)$.

Assumption 9. The forward sales, reverse recycling, cascading utilization and remanufacturing processes are all finished in a single cycle of power battery usage, which is all that the model takes into account.

3.2 Basic game relation expressions

Dealers adopt the trade-in method of recycling waste power batteries, which can not only reduce recycling costs but also promote the sales of new batteries. In order to promote the efficient use of resources through trade in, the government generally subsidizes trade in. The manufacturer's profit function is made up of the proceeds from the sale of new batteries and the expense of recycling existing batteries, based on the situations and assumptions mentioned above. Profit function for dealers is made up of sales revenue from both new and used batteries as well as the cost of recycling used batteries that are traded in by customers. The profit function of the echelon utilization user is composed of the revenue from the sale of echelon utilization products and waste batteries and the cascade utilization cost.

In the sales stage of a new power battery, the manufacturer sells the power battery ω to the distributor at a wholesale price, and the distributor sells the product p_1 to the consumer; during the waste power battery recycling process, dealers decided to provide consumers with trade-in rebates p, so as to promote consumers to return waste batteries and sell them to cascade users for cascade utilization. In the cascade utilization stage, the echelon utilizer will partially process and reorganize the recovered waste power batteries for cascade utilization and then, put them into the market for cascade usage at a cost p_t. The manufacturer, distributor and echelon utilizer are denoted by the subscripts m, s and *t*, respectively. During the analysis, the variable time *t* is removed for simplicity's sake.

In summary, the manufacturer's profit function in the infinite time domain is:

$$\pi_m = \int_0^\infty e^{-\rho t}\left\{(\omega - c_m)q_n + (\omega - c_m)q_{tn} + (c_m - c_r - f)q_{tn} - \frac{1}{2}Ax_m^2\right\}dt$$

$$= \int_0^\infty e^{-\rho t}\left\{\begin{array}{l}(\omega - c_m)N_0(1 - p_1) + (\omega - c_m)N_1\left(1 - \dfrac{p_1 - p}{1 - \delta}\right) + \\ (c_m - c_r - f)N_1\left(1 - \dfrac{p_1 - p}{1 - \delta}\right) - \dfrac{1}{2}Ax_m^2\end{array}\right\}dt \tag{1}$$

In the infinite time domain, the dealer's profit function is:

$$\pi_s = \int_0^{\infty} e^{-\rho t} \left\{ \begin{array}{l} (p_1 - \omega)q_n + (p_1 - \omega - p)q_{tn} + \\ p_2 D(p_t, n) + f(q_{tn} - D(p_t, n)) - \frac{1}{2} B x_s^2 \end{array} \right\} dt$$

$$= \int_0^{\infty} e^{-\rho t} \left\{ \begin{array}{l} (p_1 - \omega)N_0(1 - p_1) + (p_1 - \omega - p + f)N_1 \left(1 - \frac{p_1 - p}{1 - \delta}\right) \\ +(p_2 - f)(k - l p_t)n - \frac{1}{2} B x_s^2 \end{array} \right\} dt \tag{2}$$

In the infinite time domain, the echelon recycling dealer's profit function is:

$$\pi_t = \int_0^{\infty} e^{-\rho t} \left\{ (p_t - p_2 - c_1 + f)D(p_t, n) - \frac{1}{2} o x_t^2 \right\} dt$$

$$= \int_0^{\infty} e^{-\rho t} \left\{ (p_t - p_2 - c_1 + f)(k - l p_t)n - \frac{1}{2} o x_t^2 \right\} dt \tag{3}$$

Assuming that the manufacturer-led stackelberg game is adopted among manufacturer, dealer and echelon recycling dealer. The following is the expression for the Stackelberg differential model of the power battery supply chain in a closed-loop:

$$\left\{ \begin{array}{l} \max\limits_{[f, x_m]} \pi_m [f, x_m, p, x_s, p_t, x_t] \\ s.t. \left\{ \begin{array}{l} \max\limits_{[p, x_s]} \pi_s [f, x_m, p, x_s, p_t, x_t] \\ \max\limits_{[p_t, x_t]} \pi_t [f, x_m, p, x_s, p_t, x_t] \end{array} \right. \\ dn(t) = [\gamma x_m(t) + \eta x_s(t) + \alpha x_t(t) - \beta n(t)]dt \end{array} \right. \tag{4}$$

4. Equilibrium strategy

4.1 Equilibrium strategy under the unsubsidized scenario

To ascertain the manufacturer's equilibrium plan, the dealer and the echelon recycling dealer, the following analysis is carried out by using differential game theory. The power battery closed-loop supply chain's decision-making process goes like this: the manufacturer is in charge of the whole closed-loop supply chain for power batteries, which first determines the price f and recycling endeavor to recycle spent energy batteries x_m. Then the dealer determines the consumer's trade-in rebate p and trade-in recycling effort x_s.Finally, the echelon recycling dealer determines the echelon utilization price p_t both in the market for echelon usage and in its own echelon utilization initiative x_t.

Proposition 1. The best tactics for every player in the game who pursues the maximization of their own profits without subsidies are as follows:

$$f^* = \frac{-1 + \delta + 2\omega - c_r}{2} \tag{5}$$

$$p^* = \frac{-3 + 3\delta + 4p_1 - cr}{4} \tag{6}$$

$$p_t{}^* = \frac{2k - (-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)l}{4l} \tag{7}$$

$$x_m{}^* = \frac{\gamma V_m^{N'}}{A} \tag{8}$$

$$x_s{}^* = \frac{\eta V_s^{N'}}{B} \tag{9}$$

$$x_t{}^* = \frac{\alpha V_t^{N'}}{o} \tag{10}$$

Proof: Let the optimal functions of the manufacturer, the dealer and the echelon recycling dealer be V_m, V_s, V_t, respectively, then the following equation is satisfied:

$$\pi_m^* = e^{-\rho t} V_m^N;\ \pi_s^* = e^{-\rho t} V_s^N;\ \pi_t^* = e^{-\rho t} V_t^N \tag{11}$$

At any given time $t \in [0, \infty)$, the following HJB (Hamilton-Jacobi-Bellman) equations should be met by the best choice made by each player in the game, per the optimal control theory:

$$\rho V_m^N = \max\left\{\begin{array}{l} (\omega - c_m)N_0(1 - p_1) + (\omega - c_r - f)N_1\left(1 - \dfrac{p_1 - p}{1 - \delta}\right) \\ -\dfrac{1}{2}Ax_m^2 + V_m^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n) \end{array}\right\} \tag{12}$$

$$\rho V_s^N = \max\left\{\begin{array}{l} (p_1 - \omega)N_0(1 - p_1) + (p_1 - \omega - p + f)N_1\left(1 - \dfrac{p_1 - p}{1 - \delta}\right) + \\ (p_2 - f)(k - lp_t)n - \dfrac{1}{2}Bx_s^2 + V_s^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n) \end{array}\right\} \tag{13}$$

$$\rho V_t^N = \max\left\{\begin{array}{l} (p_t - p_2 - c_1 + f)(k - lp_t)n - \dfrac{1}{2}ox_t^2 \\ + V_t^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n) \end{array}\right\} \tag{14}$$

The results obtained are as follows, and the proof process is shown in the Appendix.

$$\begin{array}{l} \rho V_m^N = (\omega - c_m)N_0(1 - p_1) + \dfrac{N_1(1 - \delta - cr)^2}{8(1 - \delta)} + \\ \dfrac{\gamma^2 V_m^{N'2}}{2A} + V_m^{N'}\left(\dfrac{\eta^2 V_s^{N'}}{B} + \dfrac{\alpha^2 V_t^{N'}}{o} - \beta n\right) \end{array} \tag{15}$$

$$\rho V_s^N = (p_1 - \omega)N_0(1-p_1) + \frac{N_1(1-\delta-cr)^2}{16(1-\delta)} + \frac{CD}{8}n + \frac{\eta^2 V_s^{N'2}}{2B} + V_s^{N'}\left(\frac{\gamma^2 V_m^{N'}}{A} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n\right) \tag{16}$$

$$\rho V_t^N = \frac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)]Dn}{16l} + \frac{\alpha^2 V_t^{N'2}}{2o} + V_t^{N'}\left(\frac{\gamma^2 V_m^{N'}}{A} + \frac{\eta^2 V_s^{N'}}{B} - \beta n\right) \tag{17}$$

In which $C = 2p_2 + 1 - \delta - 2\omega + cr, D = 2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)l.$

Resolving the system of differential equations shown by Eq. (15)~Eq. (17), respectively, means finding the manufacturer's ideal value function, the dealer and the echelon recycling dealer V_m, V_s, V_t that satisfies them. Based on the problem's structure and inherent function relationship, it is presumable that the best possible roles for the maker, dealer and echelon recycling dealer are:

$$\begin{cases} V_m^N(n) = f_1 n^2 + f_2 n + f_3 \\ V_s^N(n) = i_1 n^2 + i_2 n + i_3 \\ V_t^N(n) = j_1 n^2 + j_2 n + j_3 \end{cases} \tag{18}$$

This indicates that the manufacturer's first and second derivatives of its ideal value functions, the dealer and the echelon recycling dealer are:

$$\begin{cases} V_m^{N'}(n) = 2f_1 n + f_2;\ V_m^{N''}(n) = 2f_1; \\ V_s^{N'}(n) = 2i_1 n + i_2;\ V_s^{N''}(n) = 2i_1; \\ V_t^{N'}(n) = 2j_1 n + j_2;\ V_m^{N''}(n) = 2j_1 \end{cases} \tag{19}$$

In order to make the function of the optimal value assumed in Eq. (18) be the solution of Eq. (15)~Eq. (17), the value of the coefficient $f_1, i_1, j_1, f_2, i_2, j_2, f_3, i_3, j_3$ in Eq. (18) should be determined. The detailed steps are shown in the Appendix.

It is possible to get the following result by changing the recovery effort utility of producers, distributors and cascade users into the condition of cascade utilization effort level equation:

$$dn(t) = \left[\frac{\gamma^2 V_m^{N'}}{A} + \frac{\eta^2 V_s^{N'}}{B} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n(t)\right]dt \tag{20}$$

Further obtained:

$$n(t) = \frac{25X(o\eta^2 M + B\alpha^2 N)}{Bo(3\rho+\beta)(U^2 - V)}\left(e^{\frac{3\rho+\beta}{5}t} - 1\right) + e^{\frac{3\rho+\beta}{5}t}n_0 \tag{21}$$

In which $X = \rho(1 + 2U + 2V) + \beta(2 + 3U + 3V)$

When $n_0 = 0$ and $t \to \infty$, the level of echelon utilization effort increases over time.

From the perspective of management, this represents that in the closed-loop supply chain of ladder utilization, the use of trade-in for recycling, the level of effort of ladder utilization grows rapidly with the growth of time and the strengthening of the link of interests between the decision-making subjects in the supply chain and the guarantee of the level of effort of each subject in the recycling process can help the level of effort of ladder utilization to increase more effectively, so as to realize the rational recycling of resources.

4.2 Equilibrium analysis in the context of subsidies

When the government subsidizes trade-in consumers, consumer demand for trade-in becomes $q_{tn} = N_1 \int_{\frac{p_1-p-s}{1-\delta}}^{1} 1 d\theta = N_1\left(1-\frac{p_1-p-s}{1-\delta}\right)$. The order of decision is the same as proposition one. According to the order of the game composed of the profit function, there are the following propositions:

Proposition 2. The optimal strategies for each game participant who pursues the maximization of their own profits when the government subsidizes them are:

$$f^* = \frac{-1+\delta+2\omega-c_r-s}{2} \tag{22}$$

$$p^* = \frac{-3+3\delta+4p_1-cr-3s}{4} \tag{23}$$

$$p_t{}^* = \frac{2k-(-2p_2-2c_1-1+\delta+2\omega-cr-s)l}{4l} \tag{24}$$

$$x_m{}^* = \frac{\gamma V_m^{N'}}{A} \tag{25}$$

$$x_s{}^* = \frac{\eta V_s^{N'}}{B} \tag{26}$$

$$x_t{}^* = \frac{\alpha V_t^{N'}}{o} \tag{27}$$

When subsidies are given to dealers for trade-in, trade-in consumer demand becomes $q_{tn} = N_1\left(1-\frac{p_1-p-s}{1-\delta}\right)$: At any given time $t \in [0,\infty)$, the following HJB (Hamilton–Jacobi–Bellman) equations should be met by the best choice made by each player in the game, per the optimal control theory:

$$\rho V_m^N = \max\left\{\begin{array}{l}(\omega-c_m)N_0(1-p_1)+(\omega-c_r-f)N_1\left(1-\dfrac{p_1-p-s}{1-\delta}\right)\\ -\dfrac{1}{2}Ax_m^2+V_m^{N'}(\gamma x_m+\eta x_s+\alpha x_t-\beta n)\end{array}\right\} \tag{28}$$

$$\rho V_s^N = \max\left\{\begin{array}{l}(p_1-\omega)N_0(1-p_1)+(p_1-\omega-p+f)N_1\left(1-\dfrac{p_1-p-s}{1-\delta}\right)\\ +(p_2-f)(k-lp_t)n-\dfrac{1}{2}Bx_s^2+V_s^{N'}(\gamma x_m+\eta x_s+\alpha x_t-\beta n)\end{array}\right\} \tag{29}$$

$$\rho V_t^N = \max\left\{\begin{array}{l}(p_t - p_2 - c_1 + f)(k - lp_t)n- \\ \frac{1}{2}ox_t^2 + V_t^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n)\end{array}\right\} \tag{30}$$

The reasoning process is the same as Proposition 1, and the equation of state of the level of echelon utilization effort in the case of government subsidies can be proved as follows:

$$dn(t) = \left[\frac{\gamma^2 V_m^{N'}}{A} + \frac{\eta^2 V_s^{N'}}{B} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n(t)\right]dt \tag{31}$$

Further obtained:

$$n(t) = \frac{25X(o\eta^2 M + B\alpha^2 N)}{Bo(3\rho+\beta)(U^2 - V)}\left(e^{\frac{3\rho+\beta}{5}t} - 1\right) + e^{\frac{3\rho+\beta}{5}t}n_0 \tag{32}$$

In which, $X = \rho(1 + 2U + 2V) + \beta(2 + 3U + 3V)$.

When $n_0 = 0$ and $t \to \infty$, the level of echelon utilization effort increases over time.

In which,

$U = 3\rho^2 + 8\rho\beta + 5\beta^2$, $V = 3\rho^2 + 11\rho\beta + 10\beta^2$, $N = \frac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr - s)] \times [2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - s)l]}{16l}$,

$M = \frac{(2p_2 + 1 - \delta - 2\omega + c_r + s) \cdot [2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - s)l]}{8}$.

From a managerial perspective, the level of laddering effort with subsidies changes in the same way as without subsidies, both increasing rapidly over time. The difference, however, is that when government subsidies are added, the level of laddering effort is higher than in the case without subsidies. Therefore, we can see that government subsidies can effectively promote the upgrading of the level of recycling efforts of laddering and help more waste power batteries to enter the laddering system for recycling, so as to realize the sustainable development of resources.

4.3 Equilibrium analysis in the context of rewards and punishments

When the government implements incentives and punishments for dealers, the reward and punishment expression of the dealer's profit function is added as $g(\varphi - \varphi')q_{tn}$. The order of decision is the same as proposition one. According to the order of the game composed of the profit function, there are the following propositions:

Proposition 3. The optimal strategies for each game participant who pursues the maximization of their own profits under the reward and punishment measures are as follows:

$$f^* = \frac{-1 + \delta + 2\omega - c_r - g\varphi'}{2} \tag{33}$$

$$p^* = \frac{-3 + 3\delta + 4p_1 - cr + g\varphi'}{4} \tag{34}$$

$$p_t{}^* = \frac{2k - (-2p_2 - 2c_1 - 1 + \delta + 2\omega - g\varphi' - cr)l}{4l} \tag{35}$$

$$x_m{}^* = \frac{\gamma V_m^{N'}}{A} \tag{36}$$

$$x_s{}^* = \frac{\eta V_s^{N'}}{B} \tag{37}$$

$$x_t{}^* = \frac{\alpha V_t^{N'}}{o} \tag{38}$$

Prove: Let the optimal functions of the manufacturer, the dealer and the echelon recycling dealer be V_m, V_s, V_t correspondingly; the subsequent equation is fulfilled:

$$\pi_m^* = e^{-\rho t} V_m^N;\ \pi_s^* = e^{-\rho t} V_s^N;\ \pi_t^* = e^{-\rho t} V_t^N \tag{39}$$

At any given time $t \in [0, \infty)$, the Hamilton–Jacobi–Bellman (HJB) equations are in accordance with the optimum control theory. That should be satisfied by the optimal decision of each game participant are as follows:

$$\rho V_m^N = \max\left\{\begin{array}{l}(\omega - c_m)N_0(1-p_1) + (\omega - c_r - f)N_1\left(1 - \dfrac{p_1 - p}{1-\delta}\right) - \\ \dfrac{1}{2}Ax_m^2 + V_m^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n)\end{array}\right\} \tag{40}$$

$$\rho V_s^N = \max\left\{\begin{array}{l}(p_1 - \omega)N_0(1-p_1) + (p_1 - \omega - p + f)N_1\left(1 - \dfrac{p_1 - p}{1-\delta}\right) \\ +(p_2 - f)(k - lp_t)n + g(\varphi - \varphi')N_1\left(1 - \dfrac{p_1 - p}{1-\delta}\right) - \\ \dfrac{1}{2}Bx_s^2 + V_s^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n)\end{array}\right\} \tag{41}$$

$$\rho V_t^N = \max\left\{\begin{array}{l}(p_t - p_2 - c_1 + f)(k - lp_t)n - \\ \dfrac{1}{2}ox_t^2 + V_t^{N'}(\gamma x_m + \eta x_s + \alpha x_t - \beta n)\end{array}\right\} \tag{42}$$

The reasoning process is the same as Proposition 1, and the same reason proves that under the government's reward and punishment measures, the equation of state of the level of echelon utilization effort is as follows:

$$dn(t) = \left[\frac{\gamma^2 V_m^{N'}}{A} + \frac{\eta^2 V_s^{N'}}{B} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n(t)\right] dt \tag{43}$$

Further obtained:

$$n(t) = \frac{25X(o\eta^2 M + B\alpha^2 N)}{Bo(3\rho + \beta)(U^2 - V)}\left(e^{\frac{3\rho+\beta}{5}t} - 1\right) + e^{\frac{3\rho+\beta}{5}t} n_0 \tag{44}$$

In which, $= \frac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr - g\varphi')]D}{16l} X = \rho(1 + 2U + 2V) + \beta(2 + 3U + 3V), U = 3\rho^2 + 8\rho\beta + 5\beta^2, V = 3\rho^2 + 11\rho\beta + 10\beta^2, M = \frac{CD}{8}$

When $n_0 = 0$ and $t \to \infty$ and when, the level of echelon utilization effort increases over time.

At this time, the decommissioned power batteries' cascade usage rate:

$$\varphi_{jc} = \frac{(k - lp_t)n}{q_{tn}} = \frac{n(1-\delta)[2k + (-2p_2 - 2c_1 - 1 + 2\omega - c_r - g\varphi' + \delta)l]}{N_1(1 - \delta + g\varphi' - c_r)} \tag{45}$$

When there is no subsidy, the rate at which retired power batteries are cascaded into use is:

$$\varphi_{wb} = \frac{(1-\delta)[2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r)l]n}{N_1(1 - \delta - c_r)} \tag{46}$$

When there is a subsidy, retired power batteries' echelon utilization rate is:

$$\varphi_{yb} = \frac{(1-\delta)[2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - s)l]n}{N_1(1 - \delta - c_r + s)} \tag{47}$$

All three incentives can help the level of laddering effort grow over time, but the laddering rate is higher in the subsidized case than in the other two. Therefore, from the perspective of resource sustainability, government subsidies would be more effective as government incentives than rewards and disincentives.

4.4 Comparative analysis

Table 2 shows the pairs of decision-making variables in the closed-loop supply chain of echelon utilization of trade-in power batteries without subsidies, subsidies and incentives and punishments.

Meaning of subscripts for decision variables in Table 2: yb for subsidized, wb for unsubsidized, jc for rewards and penalties.

As can be seen from Table 2, the price of recycling waste power batteries is higher than that of manufacturers under subsidies and incentives without subsidies. Under the reward and punishment model, the trade-in rebate is the highest, followed by the trade-in rebate for consumers without subsidies and the lowest trade-in rebate for consumers with subsidies. The cost of products sold by cascade users under the modes of reward and punishment is higher than that of products sold under the modes of subsidy.

The reason may be that when the government provides subsidies for dealers' trade-in behavior or implements incentives and punishments for the efficiency of recycling power batteries, to some extent, it encourages the flow of waste power batteries through the reverse supply chain. In the reward and punishment model, the incentives received by dealers are more direct than those in the subsidized mode, so the rebate provided by dealers for trade-in is higher than that in the case of subsidy. Regarding the cost of products offered by cascade users that make use of cascades, in the case of subsidies and rewards and punishments, cascade users have more opportunities to obtain waste power batteries to promote the production of cascade utilization products; therefore, in order to improve their own profits, items for cascade utilization will cost more than those for cascade utilization without subsidies.

Wi) without subsidies	With subsidies	With rewards and punishments	Comparisions
$f_{wb} = \frac{-1+\delta+2\omega-c_r}{2}$	$f_{yb} = \frac{-1+\delta+2\omega-c_r-s}{2}$	$f_{jc} = \frac{-1+\delta+2\omega-c_r-g\varphi'}{2}$	$f_{wb} > f_{yb} f_{wb} > f_{jc}$
$p_{wb} = \frac{-3+3\delta+4p_1-cr}{4}$	$p_{yb} = \frac{-3+3\delta+4p_1-cr-3s}{4}$	$p_{jc} = \frac{-3+3\delta+4p_1-cr-g\varphi'}{4}$	$p_{jc} > p_{wb} > p_{yb}$
$p_{twb} = \frac{2k-\begin{pmatrix}-2p_2-2c_1-1+\\ \delta+2\omega-cr\end{pmatrix}l}{4l}$	$p_{tyb} = \frac{2k-\begin{pmatrix}-2p_2-2c_1-1+\\ \delta+2\omega-cr-s\end{pmatrix}l}{4l}$	$p_{tjc} = \frac{2k-\begin{pmatrix}-2p_2-2c_1-1+\delta\\ +2\omega-g\varphi'-cr\end{pmatrix}l}{4l}$	$p_{tyb} > p_{twb} > p_{tjc} > p_{twb}$

Source(s): Table created by authors

Table 2. Comparative analysis of decision variables under the three measures

5. Numerical analysis

In order to analyze the influence of parameters on the best course of action for equilibrium, a study of sensitivity was performed on a series of parameters. As long as certain other factors remain constant, the government subsidy s, the customer assessment of the previous product's markdown δ, the wholesale price of the manufacturer selling the new power battery ω, the expense of remanufacturing the waste power battery c_r, the amount of the unit reward (or penalty) when the government presets the echelon utilization rate g, the recycling rate φ' set by the government under the incentive and punishment measures, the high energy density battery's unit retail price p_t that the cascade user is selling, the trade-in rebate p provided by the dealer to the consumer, the price f of the manufacturer's recycling of the waste power battery, the utility of the manufacturer's recycling efforts x_m, the results of the analysis of the impact of dealers' trade-in recycling efforts x_s and the effectiveness of cascade utilizers' efforts x_t and the conclusions, respectively, are displayed in Table 3.

Corollary 1. As government subsidies rise, the price of cascade utilization products sold by cascade users, the recycling efforts of manufacturers, distributors and cascade users have increased and the trade-in rebates provided by dealers to consumers have decreased. As a result, the cost of discarded power batteries that manufacturers recycle has dropped.

Prove:

$$\frac{\partial p_t}{\partial s} = \frac{1}{4} > 0; \ \frac{\partial p}{\partial s} = -\frac{3}{4} < 0; \ \frac{\partial f}{\partial s} = -\frac{1}{2} < 0;$$

$$\frac{\partial x_m}{\partial s} = \frac{5(U+V)}{\gamma\left(U^2-V\right)}\left[\frac{\eta^2}{8B}\left(2k+\begin{pmatrix}-4p_2-2c_1-2+2\delta\\+4\omega-2c_r-2s\end{pmatrix}l+\frac{\alpha^2 k}{2o}\right)\right] > 0;$$

$$\frac{\partial x_s}{\partial s} = \frac{5\eta(2\rho+3\beta)}{B\left(U^2-V\right)}\left[\frac{U}{8}\left(2k+\begin{pmatrix}-4p_2-2c_1-2+2\delta\\+4\omega-2c_r-2s\end{pmatrix}l\right)+\frac{BVk\alpha^2}{2o\eta^2}\right] > 0;$$

$$\frac{\partial x_t}{\partial s} = \frac{5\alpha(2\rho+3\beta)}{o\left(U^2-V\right)}\left[\begin{array}{l}\frac{U}{8}(2K+3l(-2p_2-2c_1-1+\delta+2\omega-c_r-s))+\\ \\ \frac{oV\eta^2}{8B\alpha^2}(2k+(4p_2+2c_1+2-2\delta-4\omega+2c_r+2s)l)\end{array}\right] > 0$$

Corollary 2. With the increase of the unit remanufacturing cost of waste batteries, the price of cascade utilization products sold by cascade users increases and the recycling efforts of manufacturers, distributors and cascade users increase and both the cost of discarded power batteries that manufacturers recycle

Table 3. Parameter sensitivity analysis

	p_t	p	f	x_m	x_s	x_t
s	↑	↓	↓	↑	↑	↑
c_r	↑	↓	↓	↑	↑	↑
δ	↓	↑	↑	↓	↓	↓
ω	↓	→	↑	↓	↓	↓
g	↑	↑	↓	↑	↑	↑
φ'	↑	↑	↓	↑	↑	↑

Source(s): Table created by authors

and the wholesale price of brand-new batteries that manufacturers sell drop.
Prove:

$$\frac{\partial p_t}{\partial c_r}=\frac{1}{4}>0;\ \frac{\partial p}{\partial c_r}=-\frac{1}{4}<0;\ \frac{\partial f}{\partial c_r}=-\frac{1}{2}<0;$$

$$\frac{\partial x_m}{\partial c_r}=\frac{5(U+V)}{\gamma(U^2-V)}\begin{bmatrix}\frac{\eta^2}{4B}(-2k+(4p_2+2c_1+2-2\delta-4\omega+2c_r+2s)l)+\\ \frac{\alpha^2}{8o}(2k+3(-2p_2-2c_1-1+\delta+2\omega-c_r-s)l)\end{bmatrix}>0;$$

$$\frac{\partial x_s}{\partial c_r}=\frac{5\eta(2\rho+3\beta)}{B(U^2-V)}\left[\frac{U}{8}\left(2k+\begin{pmatrix}-4p_2-2c_1-2+2\delta\\ +4\omega-2c_r-2s\end{pmatrix}l\right)+\frac{BVk\alpha^2}{2o\eta^2}\right]>0;$$

$$\frac{\partial x_t}{\partial c_r}=\frac{5\alpha(2\rho+3\beta)}{o(U^2-V)}\begin{bmatrix}\frac{U}{8}(2k+3l(-2p_2-2c_1-1+\delta+2\omega-c_r-s))+\\ \frac{oV\eta^2}{8B\alpha^2}(2k+(4p_2+2c_1+2-2\delta-4\omega+2c_r+2s)l)\end{bmatrix}>0;$$

Corollary 3. With the increase of consumer evaluation discounts, the quantity of discarded power batteries that are recycled is rising, the trade-in rebates provided by dealers to consumers and the price of waste power batteries recycled by manufacturers rises. Considering the cost at which cascade users sell goods made using cascades as well as the decline in producers', distributors' and users' recycling efforts.
Prove:

$$\frac{\partial p_t}{\partial \delta}=-\frac{1}{4}<0;\ \frac{\partial p}{\partial \delta}=\frac{3}{4}>0;\ \frac{\partial f}{\partial \delta}=\frac{1}{2}>0;$$

$$\frac{\partial x_m}{\partial \delta}=\frac{5(U+V)}{\gamma(U^2-V)}\begin{bmatrix}\frac{\eta^2}{8B}(-2k+(4p_2+2c_1+2-2\delta-4\omega+2c_r+2g\varphi')l)+\\ \frac{3\alpha^2}{8o}(-k-(-2p_2-2c_1-1+\delta+2\omega-c_r-g\varphi')l)\end{bmatrix}<0;$$

$$\frac{\partial x_s}{\partial \delta}=\frac{5\eta(2\rho+3\beta)}{B(U^2-V)}\begin{bmatrix}\frac{U}{8}(-2k-(-4p_2-2c_1-2+2\delta+4\omega-2c_r-2g\varphi')l)+\\ \frac{BV\alpha^2}{8o\eta^2}(-2k-3l(-2p_2-2c_1-1+\delta+2\omega-c_r-g\varphi'))\end{bmatrix}<0;$$

$$\frac{\partial x_t}{\partial \delta}=\frac{5\alpha(2\rho+3\beta)}{o(U^2-V)}\begin{bmatrix}\frac{U}{8}(-2k-3l(-2p_2-2c_1-1+\delta+2\omega-c_r-g\varphi'))+\\ \frac{oV\eta^2}{8B\alpha^2}(-2k-(4p_2+2c_1+2-2\delta-4\omega+2c_r+2g\varphi')l)\end{bmatrix}<0$$

Corollary 4. With the increase in the wholesale price of new power batteries sold by manufacturers, the price of waste power batteries recycled by manufacturers increases, while the price of waste power batteries sold by cascade users and the recycling efforts of manufacturers, distributors and cascade users decrease and meanwhile, it has no bearing on the trade-in discounts that dealers offer to customers.

Prove:

$$\frac{\partial p_t}{\partial \omega} = -\frac{1}{2} < 0;\ \frac{\partial p}{\partial \omega} = 0;\ \frac{\partial f}{\partial \omega} = 1 > 0;$$

$$\frac{\partial x_m}{\partial \omega} = \frac{5(U+V)}{\gamma(U^2-V)}\begin{bmatrix} \frac{\eta^2}{4B}(-2k + (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) + \\ \frac{\alpha^2}{4o}(-2k - 3(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')l) \end{bmatrix} < 0;$$

$$\frac{\partial x_s}{\partial \omega} = \frac{5\eta(2\rho+3\beta)}{B(U^2-V)}\begin{bmatrix} \frac{U}{4}(-2k - (-4p_2 - 2c_1 - 2 + 2\delta + 4\omega - 2c_r - 2g\varphi')l) + \\ \frac{BV\alpha^2}{4o\eta^2}(-2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - 2g\varphi')) \end{bmatrix} < 0;$$

$$\frac{\partial x_t}{\partial \omega} = \frac{5\alpha(2\rho+3\beta)}{o(U^2-V)}\begin{bmatrix} \frac{U}{4}(-2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')) + \\ \frac{oV\eta^2}{4B\alpha^2}(-2k - (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) \end{bmatrix} < 0$$

Corollary 5. With the increase of the amount of unit reward (or penalty) when the government presets the echelon utilization rate, the price of manufacturers to recycle waste power batteries decreases and the trade-in rebates provided by dealers to consumers, the price of echelon utilization products sold by echelon users and the recycling efforts of various entities in the supply chain increase.

$$\frac{\partial p_t}{\partial g} = \frac{1}{4} > 0;\ \frac{\partial p}{\partial c_r} = \frac{1}{4} > 0;\ \frac{\partial f}{\partial g} = -\frac{1}{2} < 0;$$

$$\frac{\partial x_m}{\partial g} = \frac{5(U+V)}{\gamma(U^2-V)}\begin{bmatrix} \frac{\eta^2}{4B}(-2k + (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) + \\ \frac{\alpha^2}{8o}(2k + 3(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')l) \end{bmatrix} > 0;$$

$$\frac{\partial x_s}{\partial g} = \frac{5\eta(2\rho+3\beta)}{B(U^2-V)}\left[\frac{U}{8}\left(2k + \begin{pmatrix} -4p_2 - 2c_1 - 2 + 2\delta \\ +4\omega - 2c_r - 2g\varphi' \end{pmatrix} l\right) + \frac{BVk\alpha^2}{2o\eta^2}\right] > 0;$$

$$\frac{\partial x_t}{\partial g} = \frac{5\alpha(2\rho + 3\beta)}{o(U^2 - V)} \begin{bmatrix} \frac{U}{8}(2k + 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')) + \\ \frac{oV\eta^2}{8B\alpha^2}(2k + (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) \end{bmatrix} > 0$$

Corollary 6. With the increase of the government's preset echelon utilization rate, the price of manufacturers to recycle waste power batteries will decrease and the trade-in rebates provided by dealers to consumers, the price of echelon utilization products sold by echelon users and the recycling efforts of various entities in the supply chain will increase accordingly.

$$\frac{\partial p_t}{\partial \varphi'} = \frac{1}{4} > 0; \ \frac{\partial p}{\partial \varphi'} = \frac{1}{4} > 0; \ \frac{\partial f}{\partial \varphi'} = -\frac{1}{2} < 0;$$

$$\frac{\partial x_m}{\partial \varphi'} = \frac{5(U + V)}{\gamma(U^2 - V)} \begin{bmatrix} \frac{\eta^2}{4B}(-2k + (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) + \\ \frac{\alpha^2}{8o}(2k + 3(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')l) \end{bmatrix} > 0;$$

$$\frac{\partial x_s}{\partial \varphi'} = \frac{5\eta(2\rho + 3\beta)}{B(U^2 - V)} \left[\frac{U}{8}\left(2k + \begin{pmatrix} -4p_2 - 2c_1 - 2 + 2\delta \\ +4\omega - 2c_r - 2g\varphi' \end{pmatrix} l \right) + \frac{BVk\alpha^2}{2o\eta^2} \right] > 0;$$

$$\frac{\partial x_t}{\partial \varphi'} = \frac{5\alpha(2\rho + 3\beta)}{o(U^2 - V)} \begin{bmatrix} \frac{U}{8}(2k + 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - c_r - g\varphi')) + \\ \frac{oV\eta^2}{8B\alpha^2}(2k + (4p_2 + 2c_1 + 2 - 2\delta - 4\omega + 2c_r + 2g\varphi')l) \end{bmatrix} > 0$$

6. Case analysis

6.1 Evolution path analysis

By simulating the assignment parameters, analyzed intuitively is the best course for the power battery closed-loop supply chain in the event of a trade-in. The parameters are set as follows (Liu and Ma, 2021): $\omega = 6$, $c_m = 4$, $c_r = 2$, $p_1 = 10$, $c_1 = 2$, $n = 0.1$, $N_0 = 0.6$, $N_1 = 0.4$, $p_2 = 5.5$, $\theta = 0.4$, $k = 35$, $\delta = 0.4$, $o = 100$, $l = 0.5$, $\rho = 0.8$, $\beta = 1$, $\alpha = 0.15$, $\gamma = 0.26$, $\eta = 0.34$, $g = 1.5$, $\varphi' = 0.2$. The trajectories of available manufacturers' profits, recyclers' profits and cascade utilizers' profits are compared under different policies, as shown in Figure 2.

As shown in Figure 3, with the growth of time, the profits of manufacturers, distributors and cascade users do not increase significantly during the period from $t = 0$ to 6, but show a rapid growth trend when t is greater than 6. At the same time, as can be observed, the echelon exploiter's profit is higher in all three scenarios, followed by the profit of the manufacturer and the profit of the dealer is the lowest. Through the simulation, it can be concluded that when the amount of unit reward (or penalty) g is greater than 25 when the government presets the echelon utilization rate under the reward and punishment system, each stakeholder's profit will exceed each subject's profit under the government subsidy.

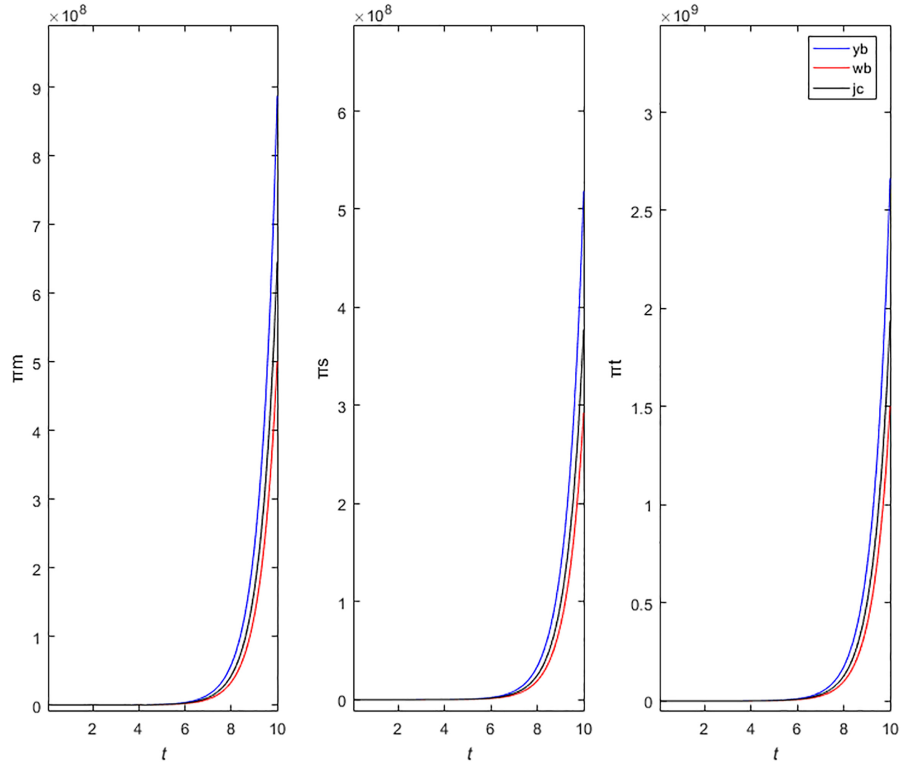

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Figure 2.
Comparison of the profits of each stakeholder under the three models

The comparative study of the closed-loop trade-in power battery supply chain's echelon utilization rate with subsidies is displayed in Figure 2, no subsidies and incentives and punishments.

As shown in Figure 3, as time goes forward, the cascade utilization rate is the highest when there is a subsidy, followed by the lowest cascade utilization rate under the incentive measures under the non-subsidy situation. The reason may lie in the fact that under the reward and punishment mode, the incentive received by the dealer is not as effective as the incentive when the subsidy is received and the subsidy is a subsidy for the trade-in behavior of the consumer, which objectively promotes the return of the consumer's battery and the reward and punishment is to take measures against the dealer from a subjective point of view; therefore, giving the customer a trade-in subsidy is the best approach to increasing the echelon's utilization rate.

6.2 Parameter sensitivity analysis

Figure 4 displays the simulation results of government subsidies and the unit remanufacturing cost of used electric vehicles, assuming that all other factors stay the same.

As shown in Figure 4, as government subsidies rise, the profits of manufacturers, distributors and cascade users and the price of cascade exploit products sold by cascade users show a linear growth trend. However, the trade-in rebate for consumers and the price of

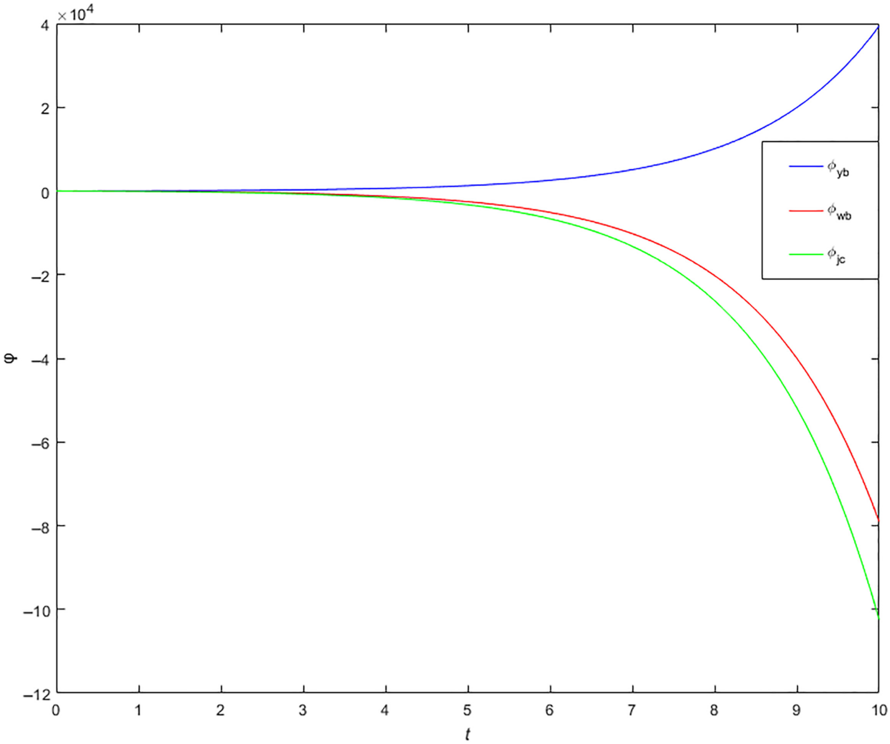

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Figure 3. Comparison of echelon utilization in the three modes

used power batteries recycled by manufacturers showed a linear downward trend with the increase of government subsidies. The cost of remanufacturing discarded power batteries has increased, the trade-in rebates provided by dealers to consumers and the prices of waste power batteries recycled by manufacturers have shown a linear growth trend, while the prices of waste power batteries sold by cascade users have shown a linear downward trend.

Figure 5 displays the simulation results of the wholesale price of the new power battery and the discount of the customer assessment of the previous product, assuming that all other factors stay the same.

As shown in Figure 5, with the increase in the utilization rate of the echelon set by the government, the price of the manufacturer's recycling of waste power batteries decreases, while the price of the echelon utilizer selling the echelon utilization product and the trade-in rebate provided by the distributor for consumers show a linear upward trend and the profits of manufacturers, distributors and echelon users show an upward trend, among which the upward trend of the manufacturer is more significant. From the perspective of wholesale prices, with the rise of the wholesale price of new power batteries, the price of cascade utilization products sold by cascade users and the profits of various entities (manufacturers, distributors and cascade users) in the supply chain have declined, while the price of manufacturers to recycle waste power batteries has shown a linear upward trend with the growth of the wholesale price of new power batteries. Changes in wholesale prices have no effect on the trade-in rebates offered by dealers to consumers.

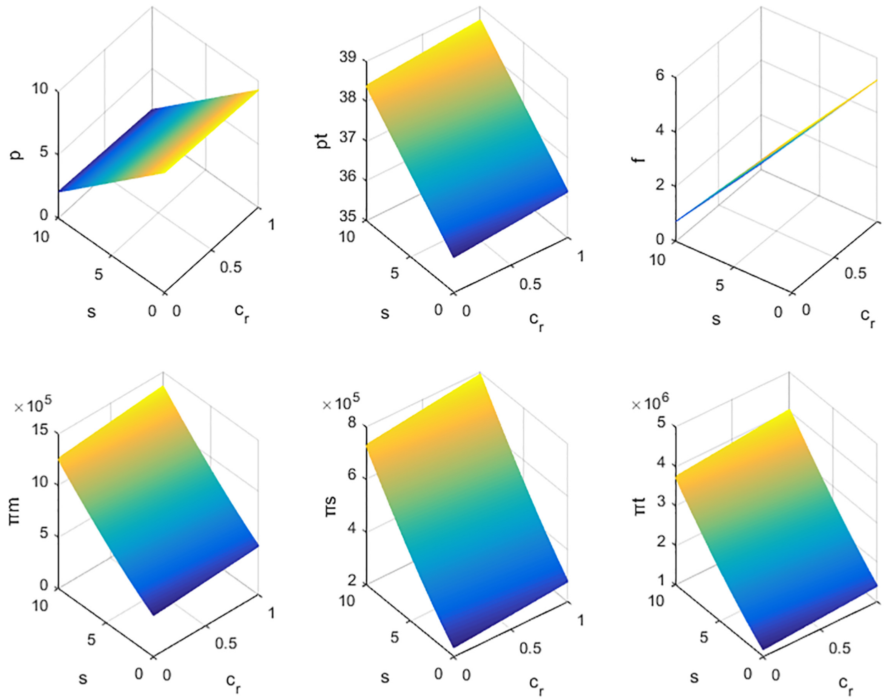

Figure 4.
Sensitivity analysis of government subsidies and unit remanufacturing costs of used power batteries

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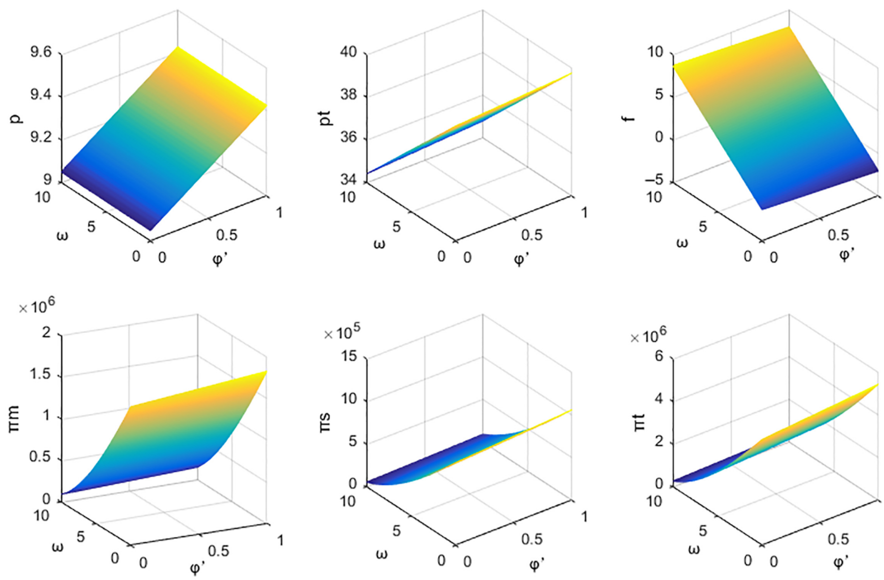

Figure 5.
Sensitivity analysis of government-set echelon utilization rate and consumers' evaluation discounts of old products under incentives and punishments

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Considering that other factors stay the same, the simulation results of consumer evaluation discounts on old products and government-set unit reward and punishment amounts are shown in Figure 6.

As shown in Figure 6, with the increase in the amount of incentives and penalties set by the government, the price of manufacturers' recycled used power batteries has shown a linear downward trend, while the profits of various entities in the supply chain (manufacturers, distributors and cascade users) have shown a linear upward trend, among which the increase of manufacturers is the most significant. The trade-in rebates provided by dealers to consumers and the sales price of cascade products sold by cascade users increase with the increase in the amount of incentives and penalties set by the government. From the perspective of consumer evaluation discounts, with the rise of consumers' evaluation discounts on old products, the trade-in rebates provided by dealers to consumers and the price of waste power batteries recycled by manufacturers show a linear growth trend, while the price of waste power batteries sold by cascade users shows a linear downward trend.

7. Conclusion

In this paper, based on differential game theory, a dual-closed-loop supply chain of power battery cascade utilization is constructed by analyzing pertinent literature and integrating it with the pertinent background of power battery echelon utilization. In the research process of this paper, the components of the power battery trade-in closed-loop supply chain are taken into account, and its organizational structure is sorted out and analyzed. The trade-in recycling method is used to more realistically and effectively integrate the power battery cascade utilization into each subject's decision-making within the closed-loop supply chain. Decision-making behaviors of each subject within the closed-loop supply chain regarding the power battery cascade utilization under no subsidy, subsidy and reward and punishment measures are also discussed, and the decision-making of each subject in the supply chain

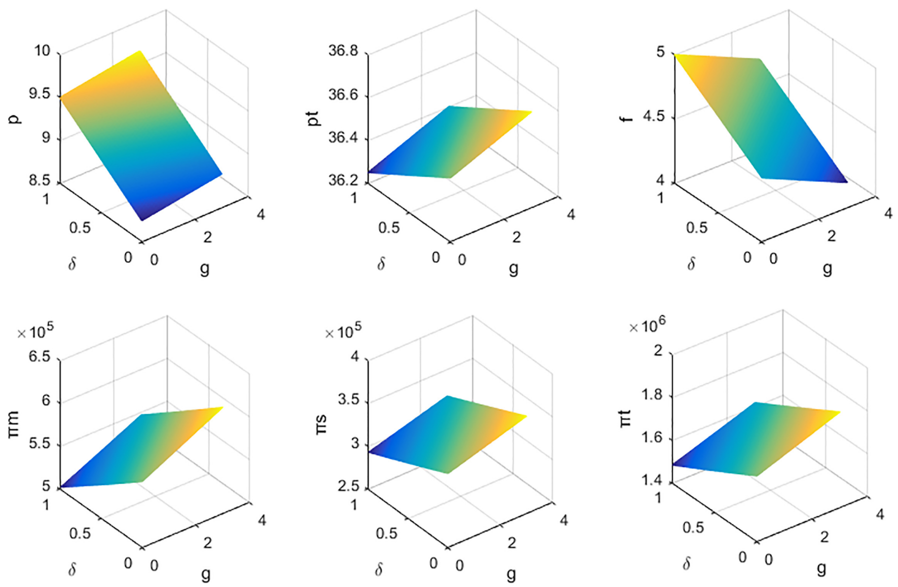

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Figure 6. Sensitivity analysis of consumers' evaluation of old products and the amount of incentives and penalties set by the government

under different measures is compared and analyzed. Finally, numerical simulation is used to carry out the visual simulation of the power battery closed-loop supply chain. This numerical simulation offers direction for the supply chain's decision-making behavior. The following elements primarily represent the paper's primary findings and accomplishments:

(1) Combining the benefits of trade-in with battery cascade use in a closed-loop supply chain, with the help of the concept of trade-in and the computational nature of consumer utility and considering the connectivity status of consumers at dealers, the trade-in scenario defines the closed-loop supply chain for power battery cascade use, fully reflecting the efficacy of trade-in in the waste power battery recycling process. Under the recycling model, the government's subsidy to consumers for trade-in behavior can help dealers reduce trade-in rebates and increase the rate at which retired power batteries are used in cascades while also assisting producers, distributors and cascade users to increase their earnings. From the consumers' point of view, the incentives and disincentives policy can be more effective in helping them to acquire new power cells at lower prices. Therefore, the utilization rate of the government's trade-in subsidy and the incentives and penalties set by it should be controlled within a reasonable range to protect the rights and interests of consumers and all entities in the supply chain.

(2) According to the trade-in link, the government's subsidy helps the dealers to reduce the trade-in rebates provided, thereby helping dealers reduce the loss caused by trade-in and promote the increase of dealers' overall profits. The smaller the discount of consumers' evaluation of old products, the more conducive to the increase of profits of various stakeholders in the supply chain under the three scenarios. Under the combined effect of consumers' evaluation discounts on old products and the amount of rewards and punishments set by the government, the profit growth of manufacturers is the largest and the profit growth of dealers is the smallest. Therefore, government subsidies are more effective than incentives and punishments in the incentive of the profits of various entities in the supply chain.

(3) Regarding the echelon application of decommissioned power sources, improving the echelon utilization rate not only depends on government subsidies but also is closely related to the price of echelon utilization products sold by echelon users and the trade-in interest rebate provided by dealers to consumers. Subsidizing the recycling method of trade-in in the supply chain can help consumers improve the retrospective of old products, promote the liquidity between various game players and promote more waste power batteries to enter the cascade utilization cycle process and improve the cascade utilization rate. Under the reward and punishment measures, a win-win situation can be achieved by raising the government's set echelon utilization rate, which will also effectively encourage an increase in the earnings of all supply chain participants. It is evident that government subsidies and incentives can raise echelon users', distributors' and manufacturers' profits while also somewhat increasing the echelon utilization rate. However, it is necessary to control the echelon utilization rate preset by the government within a reasonable range to ensure the effectiveness of incentives.

In order to the sustainable development of resources and effective protection of the environment, the stepwise utilization of waste power batteries cannot be delayed. From a long-term perspective, the government subsidies can not only effectively help the supply chain of the main body to obtain the growth of profits but also incentivize more waste power batteries to enter the stepwise utilization system for the recycling of resources. The model constructed in this paper has a sustainable impact in two main ways: firstly, the government's incentive policies have an economic promoting effect on the closed-loop recycling and reuse of power batteries. Both government subsidies and reward and punishment policies help waste power batteries enter the closed-loop supply chain, promoting the profits of the three main decision-makers and facilitating sustainable environmental development. Secondly, replacing old batteries with new ones will be the main recycling mode for power batteries, directly recycling waste power batteries from consumers, which to some extent saves resources and reduces environmental carrying capacity. How to reasonably utilize waste power batteries and protect unexplored metal resources is a key issue in achieving sustainable development.

References

Cao, K.Y., Bo, Q.S. and He, Y. (2018), "Optimal trade-in and third-party collection authorization strategies under trade-in subsidy policy", *Kybernetes: The International Journal of Systems and Cybernetics*, Vol. 47 No. 5, pp. 854-872, doi: 10.1108/k-07-2017-0254.

Desai, P.S., Purohit, D. and Zhou, B. (2016), "The strategic role of exchange promotions", *Marketing Science*, Vol. 35 No. 1, pp. 93-112, doi: 10.1287/mksc.2015.0955.

Gong, B.G., Gao, Y.L. and Liu, Z. (2018), "Selection of recycling channels for power battery closed-loop supply chain under government fund policies", *Computer Integrated Manufacturing Systems*, Vol. 24 No. 2, pp. 1-19.

Gu, X.Y., Ieromonachou, P., Zhou, L. and Tseng, M.L. (2018), "Developing pricing strategy to optimise total profits in an electric vehicle battery closed loop supply chain", *Journal of Cleaner Production*, Vol. 203, pp. 376-385, doi: 10.1016/j.jclepro.2018.08.209.

Han, X.H., Yang, Q.X., Shang, J. and Pu, X. (2017), "Optimal strategies for trade-old-for-remanufactured programs: receptivity, durability, and subsidy", *International Journal of Production Economics*, Vol. 193, pp. 602-616, doi: 10.1016/j.ijpe.2017.07.025.

Harper, G., Sommervill, R., Kendrick, E., Driscoll, L., Slater, P., Stolkin, R., Walton, A., Christensen, P., Heidrich, O., Lambert, S., Abbott, A., Ryder, K., Gaines, L. and Anderson, P. (2019), "Recycling lithium-ion batteries from electric vehicles", *Nature*, Vol. 575 No. 7781, pp. 75-86, doi: 10.1038/s41586-019-1682-5.

Huang, Y.T. (2018), "A closed-loop supply chain with trade-in strategy under retail competition", *Mathematical Problems in Engineering*, Vol. 1 No. 2018, pp. 1-16, doi: 10.1155/2018/1510959.

Jiao, J., Pan, Z. and Li, J. (2022), "Effect of carbon trading scheme and technological advancement on the decision-making of power battery closed-loop supply chain", *Environmental Science and Pollution Research*, Vol. 2022 No. 6, pp. 1-22, doi: 10.1007/s11356-022-23078-6.

Kang, D.H.P., Chen, M. and Ogunseitan, O.A. (2013), "Potential environmental and human health impacts of rechargeable lithium batteries in electronic waste", *Environmental Science and Technology: ES&T*, Vol. 47 No. 10, pp. 5495-5503, doi: 10.1021/es400614y.

Ke, H. and Cai, M.S. (2018), "Pricing decisions in closed-loop supply chain with collection-relevant demand", *Journal of Remanufacturing*, Vol. 9 No. 2, pp. 75-87, doi: 10.1007/s13243-018-0060-z.

Li, Z., Zheng, W., Meng, Q.F. and Jin, S. (2018), "The impact of government subsidy and tax policy on the competitive decision-making of remanufacturing supply chains", *International Journal of Sustainable Engineering*, Vol. 12 No. 1, pp. 18-29, doi: 10.1080/19397038.2018.1449031.

Liu, J.J. and Guo, Y.K. (2021), "Design of reverse logistics network for electric vehicle powerbattery considering uncertainty", *Journal of Shanghai Maritime University*, Vol. 42 No. 2, pp. 96-102.

Liu, J.J. and Ma, J.L. (2021), "Research on reverse subsidy mechanism of power battery closed-loop supply chain considering cascade utilization", *Industrial Engineering and Management*, Vol. 26 No. 3, pp. 80-88.

Liu, J.W., Hou, Q. and Bao, W.T. (2023), "A closed-loop supply chain game model considering manufacturers' fair concern and cascade utilization", *Systerms Engineering*, Vol. 41 No. 2, pp. 89-99.

Ma, W.M., Zhao, Z. and Ke, H. (2013), "Dual-channel closed-loop supply chain with government consumption-subsidy", *European Journal of Operational Research*, Vol. 226 No. 2, pp. 221-227, doi: 10.1016/j.ejor.2012.10.033.

Miao, Z.W., Fu, K., Xia, Z.Q. and Wang, Y. (2017), "Models for closed-loop supply chain with trade-ins", *Omega*, Vol. 66, pp. 308-326, doi: 10.1016/j.omega.2015.11.001.

Mu, D., Yang, J. and Li, X. (2021), "The impact of closed-loop supply chain enterprise cooperation on the recycling and reuse of new energy vehicle power batteries", *Supply Chain Management*, Vol. 2 No. 1, pp. 54-67.

Ordonez, J., Gago, E.J. and Girard, A. (2016), "Processes and technologies for the recycling and recovery of spent lithium-ion batteries", *Renewable and Sustainable Energy Reviews*, Vol. 60, pp. 195-205, doi: 10.1016/j.rser.2015.12.363.

Ramoni, M.O. and Zhang, H.C. (2013), "End-of-life (EOL) issues and options for electric vehicle batteries", *Clean Technologies and Environmental Policy*, Vol. 15 No. 6, pp. 881-891, doi: 10.1007/s10098-013-0588-4.

Shu, T., Huang, C.F., Chen, S., Wang, S. and Lai, K.K. (2018), "Trade-Old-for-Remanufactured closed-loop supply chains with carbon tax and government subsidies", *Sustainability*, Vol. 10 No. 11, pp. 1-25, doi: 10.3390/su10113935.

Tian, H.P. and Liu, C.X. (2011), "Biobjective incentive model in distribution channel under the framework of dual information asymmetry", *Journal of Management Science in China*, Vol. 14 No. 3, pp. 34-47.

Umangi, P., Ravi, K. and Ravi, S. (2020), "Effect of buyback price on channel's decision parameters for manufacturer-led close loop dual supply chain", *Opsearch*, Vol. 57 No. 2, pp. 438-461, doi: 10.1007/s12597-019-00421-z.

Webster, S. and Mitra, S. (2007), "Competitive strategy in remanufacturing and the impact of take-back laws", *Journal of Operations Management*, Vol. 25 No. 6, pp. 1123-1140, doi: 10.1016/j.jom.2007.01.014.

Xiao, L., Wang, X., Chin, K.S. and Qin, Y. (2017), "Competitive strategy in remanufacturing and the effects of government subsidy", *Journal of Systems Science and Systems Engineering*, Vol. 26 No. 4, pp. 417-432, doi: 10.1007/s11518-017-5345-5.

Zeng, X.L., Li, J.H. and Singh, N. (2014), "Critical reviews in environmental science and technology recycling of spent lithium-ion battery: a critical review recycling of spent lithium-ion battery: a critical review", *C R C Critical Reviews in Environmental Control*, Vol. 44 No. 10, pp. 1129-1165, doi: 10.1080/10643389.2013.763578.

Zeng, X.L., Li, J.H. and Liu, L.L. (2015), "Solving spent lithium-ion battery problems in China: opportunities and challenges", *Renewable and Sustainable Energy Reviews*, Vol. 52, pp. 1759-1767, doi: 10.1016/j.rser.2015.08.014.

Zhang, C. and Chen, Y.X. (2021), "Decision and coordination of cascade utilization power battery closed-loop supply chain with economies of scale under government subsidies", *Operations Research and Management Science*, Vol. 30 No. 12, pp. 72-77.

Zhang, F.A., Li, N. and Da, Q.L. (2022), "Research on low-carbon emission reduction in a closed-loop supply chain with multiple demands under two subsidy policies", *Chinese Journal of Management Science*, Vol. 31 No. 10, pp. 116-127.

Zhao, S.L., Zhu, Q.H. and Cui, L. (2016), "A decision-making model for remanufacturers: considering both consumers' environmental preference and the government subsidy policy", *Resources Conservation and Recycling*, Vol. 128, pp. 176-186, doi: 10.1016/j.resconrec.2016.07.005.

Zhu, X.D. and Yu, L.F. (2019), "Screening contract excitation models involving closed-loop supply chains under asymmetric information games: a case study with new energy vehicle power battery", *Applied Sciences*, Vol. 9 No. 1, p. 146, doi: 10.3390/app9010146.

Zhu, X.X., Wang, M.M., Chen, G.F. and Chen, X. (2016), "The effect of implementing trade-in strategy on duopoly competition", *European Journal of Operational Research*, Vol. 248 No. 3, pp. 856-868, doi: 10.1016/j.ejor.2015.07.053.

Appendix

This section is dedicated to the analysis and proof of Propositions 1.

(1) The inverse induction method is applied to resolve the model, firstly, the optimal decision of the cascade user is solved and the derivative of the echelon utilization price and the recovery effort of the cascade utilizer are, respectively, obtained.

$$\frac{\partial \rho V_t^N}{\partial p_t} = (k - lp_t)n - (p_t - p_2 - c_1 + f)n \tag{1}$$

$$\frac{\partial \rho V_t^N}{\partial x_t} = -ox_t + V_t^{N'}\alpha \tag{2}$$

Let the Eq. (1) and (2) be equal to 0, the reaction function are calculated of p_t and x_m to p, x_s, respectively, at the same time, bring them into the dealer's profits and ask for the derivation:

$$\frac{\partial \rho V_s^N}{\partial p} = -N_1\left(1 - \frac{p_1 - p}{1 - \delta}\right) + (p_1 - \omega - p + f)\frac{N_1}{1 - \delta} \tag{3}$$

$$\frac{\partial \rho V_s^N}{\partial x_s} = -Bx_s + V_s^{N'}\eta \tag{4}$$

Let the Eq. (3) and (4) be equal to 0, p and x_s the reaction function are calculated p, x_s to f, x_m, respectively. What will be obtained to be entered in the manufacturer's profit, and the derivative will be obtained:

$$\frac{\partial \rho V_m^N}{\partial f} = -N_1\frac{3 - 3\delta - 2p_1 - \omega - f}{2(1 - \delta)} - (\omega - c_r - f)N_1\frac{1}{2(1 - \delta)} \tag{5}$$

$$\frac{\partial \rho V_m^N}{\partial x_m} = -Ax_m + \gamma V_m^{N'} \tag{6}$$

Let the Eq. (5) and (6) be equal to 0, and find the optimal f, x_m, bring f, x_m, back to the reaction function of the p, x_s, p_t, x_m and find the manufacturer's ideal course of action, distributor and cascade user as proposition one, which is completed.

The results obtained are as follows:

$$\rho V_m^N = (\omega - c_m)N_0(1 - p_1) + \frac{N_1(1 - \delta - cr)^2}{8(1 - \delta)} + \frac{\gamma^2 V_m^{N'2}}{2A} + V_m^{N'}\left(\frac{\eta^2 V_s^{N'}}{B} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n\right) \tag{7}$$

$$\rho V_s^N = (p_1 - \omega)N_0(1 - p_1) + \frac{N_1(1-\delta-cr)^2}{16(1-\delta)} + \frac{CD}{8}n + \frac{\eta^2 V_s^{N'2}}{2B} + V_s^{N'}\left(\frac{\gamma^2 V_m^{N'}}{A} + \frac{\alpha^2 V_t^{N'}}{o} - \beta n\right) \tag{8}$$

$$\rho V_t^N = \frac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)]Dn}{16l} + \frac{\alpha^2 V_t^{N'2}}{2o} + V_t^{N'}\left(\frac{\gamma^2 V_m^{N'}}{A} + \frac{\eta^2 V_s^{N'}}{B} - \beta n\right) \tag{9}$$

In which $C = 2p_2 + 1 - \delta - 2\omega + cr, D = 2k + (-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)l.$

(1) So, substituting Eq. (18) and Eq. (19) into Eq. (15) ~ Eq. (17), respectively, obtains:

$$\begin{cases} \rho f_1 = \dfrac{2\gamma^2 f_1^2}{A} + 2f_1\left(\dfrac{2\eta^2 i_1}{B} + \dfrac{2\alpha^2 j_1}{o} - \beta\right) \\ \rho f_2 = \dfrac{2\gamma^2 f_1 f_2}{A} + 2f_1\left(\dfrac{\eta^2 i_2}{B} + \dfrac{\alpha^2 j_2}{o}\right) + f_2\left(\dfrac{2\eta^2 i_1}{B} + \dfrac{2\alpha^2 j_1}{o} - \beta\right) \\ \rho f_3 = (\omega - c_m)N_0(1 - p_1) + \dfrac{N_1(1-\delta-cr)^2}{8(1-\delta)} - \dfrac{\gamma^2 f_2^2}{2A} \end{cases} \tag{10}$$

$$\begin{cases} \rho i_1 = 2i_1\left(\dfrac{2\gamma^2 f_1}{A} + \dfrac{2\alpha^2 j_1}{o} - \beta\right) \\ \rho i_2 = \dfrac{CD}{8} + \dfrac{2i_1 i_2 \eta^2}{B} + 2i_1\left(\dfrac{\gamma^2 f_2}{A} + \dfrac{\alpha^2 j_2}{o}\right) + i_2\left(\dfrac{2\gamma^2 f_2}{A} + \dfrac{2\alpha^2 j_2}{o} - \beta\right) \\ \rho i_3 = (p_1 - \omega)N_0(1 - p_1) + \dfrac{N_1(1-\delta-cr)^2}{16(1-\delta)} - \dfrac{\eta^2 i_2^2}{2B} \end{cases} \tag{11}$$

$$\begin{cases} \rho j_1 = 2j_1\left(\dfrac{2\gamma^2 f_1}{A} + \dfrac{2\eta^2 i_1}{B} - \beta\right) \\ \rho j_2 = \dfrac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)]D}{16l} + \dfrac{2j_1 j_2 \alpha^2}{o} + \\ 2j_1\left(\dfrac{\gamma^2 f_2}{A} + \dfrac{\eta^2 i_2}{B}\right) + j_2\left(\dfrac{2\gamma^2 f_1}{A} + \dfrac{2\eta^2 i_1}{B} - \beta\right) \\ \rho j_3 = -\dfrac{\alpha^2 j_2^2}{2o} \end{cases} \tag{12}$$

Eq. (10)~Eq. (12) contain every potential value for the cascade utilization effort level. This indicates that the coefficients and constant terms of the total on both sides must be equal.

The coefficients of the optimal function are $f_1, i_1, j_1, f_2, i_2, j_2, f_3, i_3, j_3$ as follows:

$$\begin{cases} f_1 = \dfrac{(\rho + 2\beta)A}{10\gamma^2} \\ f_2 = \dfrac{5A(U+V)(\rho+2\beta)\left(\dfrac{\eta^2 M}{B} + \dfrac{\alpha^2 N}{o}\right)}{\gamma^2 (U^2 - V)} \\ f_3 = \dfrac{1}{\rho}\left[(\omega - c_m)N_0(1-p_1) + \dfrac{N_1(1-\delta-cr)^2}{8(1-\delta)}\right] - \\ \dfrac{25(U+V)^2(\rho+2\beta)^2\left(\dfrac{\eta^2 M}{B} + \dfrac{\alpha^2 N}{o}\right)^2}{2\rho\gamma^2(U^2-V)^2} \end{cases} \tag{13}$$

$$\begin{cases} j_1 = \dfrac{(\rho + 2\beta)o}{10\alpha^2} \\ j_2 = \dfrac{5B\alpha^2 NU(2\rho+3\beta) + 5o\eta^2 MV(2\rho+3\beta)}{B\alpha^2(U^2-V)} \\ j_3 = -\dfrac{\alpha^2}{2\rho o}\left(\dfrac{5B\alpha^2 NU(2\rho+3\beta) + 5o\eta^2 MV(2\rho+3\beta)}{B\alpha^2(U^2-V)}\right)^2 \end{cases} \tag{14}$$

$$\begin{cases} i_1 = \dfrac{(\rho + 2\beta)B}{10\eta^2} \\ i_2 = \dfrac{5MUo\eta^2(2\rho+3\beta) + 5NVB\alpha^2(2\rho+3\beta)}{o\eta^2(U^2-V)} \\ i_3 = \dfrac{1}{\rho}\left[(p_1 - \omega)N_0(1-p_1) + \dfrac{N_1(1-\delta-cr)^2}{16(1-\delta)} - \right] - \\ \dfrac{\eta^2}{2\rho B}\left(\dfrac{5MUo\eta^2(2\rho+3\beta) + 5NVB\alpha^2(2\rho+3\beta)}{o\eta^2(U^2-V)}\right)^2 \end{cases} \tag{15}$$

In which, $U = 3\rho^2 + 8\rho\beta + 5\beta^2$, $V = 3\rho^2 + 11\rho\beta + 10\beta^2$, $M = \frac{CD}{8}$, $N = \frac{[2k - 3l(-2p_2 - 2c_1 - 1 + \delta + 2\omega - cr)]D}{16l}$

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