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Practicing variation theory beyond learning study

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Abstract

Purpose – The purpose of this paper is to explore whether and how principles from variation theory can contribute to the planning of teaching and learning beyond learning study.

Design/methodology/approach – We study whether and how principles from variation theory contributed to a group of teachers' planning of teaching and learning about decimal numbers in Grades 4 to 7 working in Subject Didactic Groups – a collaborative arrangement suited to daily teaching. A theoretical thematic analysis approach was used when analyzing eight audio-recorded meetings and written documents.

Findings – The study shows that variation theory principles contributed to the teachers' planning of teaching and learning. Two themes were identified: the theory contributed to the teachers being able to (1) specify what their students needed to learn and (2) design tasks that they anticipated would afford the opportunity to learn what was identified as being necessary to learn.

Originality/value – The paper demonstrates how variation theory can contribute to teachers' planning of teaching and learning when used in a collaborative arrangement other than learning study. This leads into a discussion about variation theory being used separately from learning study and the benefits and limitations this other collaborative arrangement can have for gaining knowledge of what is to be learned and taught.

Keywords Variation theory of learning, Learning study, Critical aspects, Subject didactic groups, Collaborative inquiry, Decimal number addition

Paper type Research paper

Introduction

In several publications, in this and other journals, it has been demonstrated how variation theory (VT) (Marton, 2015) can contribute to student learning (e.g. Lo *et al.*, 2005; Ting *et al.*, 2018), and teacher learning (e.g. Holmqvist, 2011; Yok, 2012), in the context of learning study (LearS) (Pang and Marton, 2003). In almost all LearS, VT has been the framing theory. However, it has been suggested that: "It could be seen as a problem that variation theory and learning study are seen as almost intertwined, however, and that learning study has become more or less a synonym to variation theory" (Runesson, 2016, p. 295).

Runesson argues that both VT and LearS need to be able to stand alone. LearS must be opened up for a broader theoretical framing, just as VT must be applied beyond the borders of LearS. It has been demonstrated that theories other than VT can frame LearS. For instance, theories from mathematics education were used by Martin and Towers (2016) ("folding back", Pirie and Kieren, 1994) and by Eriksson and Sumpter (2021) (activity theory, Davydov, 2008). However, there are few studies that use VT independently of

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LearS. Our study aims to contribute to the understanding of the role and significance of VT in teachers' collaborative work beyond LearS.

Taking inspiration from other forms of collaborative work (lesson study, learning study, and Chinese Teaching Research Groups), an arrangement—Subject Didactic Group (SDG)—was developed by a group of researchers and teachers at the University of Gothenburg in Sweden. SDG has been implemented in approximately 70 schools in Sweden over the last 10 years. The overall aim of SDG is to create a sustainable culture of collaborative work to enhance student learning on a daily basis in a school. Just as in LearS, in SDG the process is guided by VT. This has specific implications. VT asserts that learning is a change in how a phenomenon is experienced, and learning difficulties are explained in terms of a failure to discern aspects that are critical for learning (Marton, 2015). Discernment presupposes an experienced variation of an aspect, and that which is varied is likely to be discerned. To identify what students have not (yet) discerned (the critical aspect), the teacher must understand how her students experience that which is to be learned. In VT, this entails adopting a *second-order perspective*. To afford opportunities to discern the critical aspect, a systematic *variation* against a background of *invariance* must be provided and be possible to experience. In this study, we explore the role of VT in the SDG by studying one teacher group planning a series of lessons and evaluating student learning of decimal numbers.

Given the above, we pose the following research question:

> In what ways can principles of VT, namely *second-order perspective* and *variation/invariance,* contribute to teachers' planning of teaching and learning when participating in Subject Didactic Groups?

Contribution of VT in LearS

The benefits of applying VT in LearS have been pointed out previously (Elliott, 2012; Kullberg *et al.*, 2024; Pang and Marton, 2003). Specifically, studies have shown that LearS, in which teachers are guided by VT in planning and assessment, can contribute to enhancing student learning. In a quasi-experimental study, Pang and Marton (2003) examined the relationship between student learning and teaching by comparing two ways of handling the same object of learning. One of the teacher groups in the study planned and enacted a series of lessons based on teaching experiences and VT principles: The other teacher group did not use these principles, just their teaching experiences when planning and teaching their lessons. The conclusion drawn, was that the differences in learning outcomes—higher in the class in which the teachers were guided by VT than in the other class—depend on differences in how the content was handled in the different classrooms. Kwok and Chik (2005) investigated the progress of student learning in 27 LearS and showed that the increase in students' results between pre- and post-tests was statistically significant for both high- and low-achieving students. Several other studies have also demonstrated the potential of applying VT for enhancing student learning in different subjects and contexts (e.g. Holmqvist *et al.*, 2012; Ting *et al.*, 2018).

Why is it that VT seems to guide teachers to teach in a powerful way that enhances students' learning? According to Marton (2015), it is because the theory can give teachers access to knowledge about the necessary conditions for learning.

Necessary conditions for learning

From a VT perspective, one necessary condition for learning is being able to discern new aspects of what is being learned. Discerning new aspects and relating them to other aspects implies a change in how something is experienced (Marton, 2015). Hence, to provide learning opportunities and help the learners to experience the object of learning in the targeted way, it

is necessary to identify and teach the critical aspects for learning. This conjecture has been tested and is supported by several studies (e.g. Kullberg *et al.*, 2016a; Nilsson, 2014; Wood *et al.*, 2016). From these studies it can be concluded that VT *per se* does not stipulate what the critical aspects are or how they should be handled in teaching. However, VT can work as a pedagogical tool that can challenge the teacher's understanding and beliefs about teaching, which provides a good basis for identifying aspects critical for student learning.

In order to be able to identify the critical aspects, teachers need to take a second-order perspective on the object of learning (Pang and Ki, 2016). This implies making statements about the world as experienced by others. Hence, teachers must try to understand the significance of students' answers, rather than just seeing them as incorrect or as misinterpretations. Several studies show that this is possible in the context of LearS. Teachers can shift their way of seeing the object of learning, from their own perspective to the students' perspective (e.g. Elliot and Yu, 2013; Holmqvist, 2011), and such a shift plays a vital role for teachers in understanding more clearly what the critical aspects are, which in turn affects their teaching.

Another necessary condition for learning is that discernment requires the experience of variation against a background of invariance (Marton, 2015). If something varies against a stable background, it is likely to be discerned. Having once identified the critical aspects of the object of learning, the teacher can be guided as to how to provide opportunities for discernment by opening up variation of the critical aspect while keeping other aspects invariant (e.g. Kullberg *et al.*, 2016b; Pang and Lo, 2012; Holmqvist, 2011). Studies have explicitly shown that in the collaborative work in LearS, principles from VT can guide teachers in planning and teaching mathematics tasks with specific patterns of variation and in that way draw students' attention to specific critical aspects, for instance, juxtaposing and comparing, thus making a contrast (Pillay *et al.*, 2022), or sequencing and pairing examples with systematic variation within as well as between examples (Watson and Mason, 2006).

To conclude, teachers have access to knowledge about the necessary conditions for learning when guided by VT in planning and evaluating teaching in LearS. This presupposes using two principles of VT: (1) adopting a *second-order perspective* to identify the critical aspects for learning and (2) plan to make these discernible by means of *variation against a background of invariance*.

VT in contexts beyond LearS

There are few studies that focus on what VT can contribute to teaching and learning, independently of LearS. What teachers can learn from being introduced to VT when participating in a nationwide program for teacher professional development in teaching mathematics has been studied (Olteanu, 2016). The study showed that VT contributed to teachers' decision-making through the teachers' reflection on what their students needed to learn, in terms of critical aspects, and how the content could be handled, in terms of patterns of variation. It was also shown that VT contributed to the teachers' reflection on the relation between the intended student learning and what the students learned.

However, neither this national professional development program nor LearS have as their main aim to support and integrate principles from VT in teachers' daily teaching and thereby into the school culture. LearS is often a specific project lasting for shorter periods of time (Cheng and Lo, 2013). It is rather time-consuming, and needs additional economic and other resources, and it may therefore be difficult to continue with the fundamentals of LearS when the project has ended. As described earlier, this study aims to investigate what VT can contribute to teaching beyond LearS when used with the aim of establishing a collaborative

learning culture (SDG) among teachers in a school that can support teachers' daily teaching. In order to better understand SDG, it is helpful to compare it to LearS.

LearS and SDG, similarities and differences

LearS is a hybrid of Japanese lesson study (Lewis and Tsuchida, 1999) and design experiments (Brown, 1992), and can be characterized as an "ongoing and dialectical exploration of practice and theory" (Tan *et al.*, 2020, p. 443). LearS can be used for the purpose of research (Carlgren, 2020), testing theories (Pang and Marton, 2003), and teacher professional development (Elliot and Yu, 2013). In the 2010s, a group connected to the University of Gothenburg in Sweden, with extensive experience of LearS, concluded that LearS seldom becomes implemented as a lasting culture at schools. With the aim of creating a sustainable and collaborative community that supports teachers' daily and continuous teaching, the group drew inspiration from LearS but also the Japanese lesson study and the weekly activities of lesson planning and assessment of student learning that take place in Chinese Teaching Research Groups (Paine and Ma, 1993). In 2014, SDG was tried out as a pilot project in Sweden (Mårtensson, 2020).

As previously pointed out, VT has played a pivotal role in guiding teachers in their planning for learning in LearS, that is, to identify the critical aspects and how to use principles of variation and invariance to make them discernible for their students. Hence, the planning activities are very much focused on how to strengthen the link between instruction and student learning of specific objects of learning. In this respect, LearS is similar to SDG, although the nature of the object of learning differs. In a LearS, the object of learning is explored through the systematic work of planning, analyzing, and revising a single lesson recurrently to capture the critical aspects (Marton, 2015). It has been shown that such exploration also can guide teachers to make delimitations and distinctions regarding the object of learning and that this affects teaching (Kullberg *et al.*, 2016a). As the aim with SDG is to make sustained collaborative inquiry part of teachers' daily work, the object of learning is not explored in the same way. Therefore, the characteristics are more in line with a learning objective that describes more generally what students should be able to do or learn. Pre-and post-tests are used formatively, both in LearS and SDG, to provide information about student understanding and thereby identify critical aspects. But as learning objectives are formulated in more general terms in SDGs, this could affect the success (or failure) of identifying the critical aspects. Therefore, it is important that teachers in SDGs also use their lessons to notice indications of students' understanding, for example from students' written answers, students' answers to the teacher's questions, and discussions among students. Such complementary data is taken up for discussion in the meetings to identify student learning and new critical aspects for upcoming lessons.

Since the aim of SDG is to create a sustainable collaborative culture for daily teaching, the way it is organized differs from LearS in some respects. First, a common feature of LearS is that a researcher or an external tutor has an active role in the group, for instance in sharing research findings, analyzing pre- and post-tests, giving expert feedback, and leading discussions around lessons using a theory of learning (Lo *et al.*, 2005). Instead of being researcher-led, SDGs are teacher-led. In each group, there is an ordinary teacher who has the role of an SDG leader. These teacher leaders are commonly trained in how to support collaboration and how to challenge the group members when jointly planning and evaluating their daily teaching, for example by using VT principles (Mårtensson, 2020). However, SDG leaders do not have the autonomy to make decisions on what and how to teach, nor are they frontline teachers, who demonstrate teaching for others, which are common characteristics among teacher leaders in other forms of professional learning communities (Shan and Chen, 2022). Decisions about instruction plans, teaching approaches, tasks to use etcetera are taken

by the group members. A second difference is the time allocated for teachers to collaborate. Teachers in SDG usually meet regularly for about 1–1.5 h a week throughout the whole school year (Mårtensson, 2020), which probably plays a key role in promoting the formation of a lasting collaboration community at the school. In LearS, the group members often meet only for a more limited period of time. Third, the cyclic and iterative process of revising a single lesson, with different groups of students, that takes place in LearS is not present in SDG. This implies that the teachers do not film or observe each other's lessons. Even though observations, for example open lessons (e.g. Miyakawa and Winsløw, 2013), can be included in SDG, it is not mandatory due to time constraints. Instead, the teachers teach their own classes in parallel and link each lesson to the next based on joint discussions and reflections on previous lessons. These reflections are used to collaboratively plan upcoming lessons.

Method

Context of the study

The present study was conducted at a Swedish elementary school, selected as the teachers there had experience of SDG. They had been working in SDGs for about two years and there were not many other schools with this level of experience. To select one teacher group to follow and collect data from, an information-oriented selection (Flyvbjerg, 2006) was used, that is, we expected (based on a discussion with the principal at the school) that the selected SDG could most likely maximize the information obtained and highlight prototypical characteristics for our study. The five teachers in the selected SDG gave their consent to participate. They had 1.5–40 years of teaching experience and were teaching mathematics in Grades 4 to 7—two of them taught mathematics as their main subject (Grade 7) and the others taught several subjects including mathematics (Grades 4 to 6). Half a year before the present study, these teachers had formed their SDG and they had been meeting regularly since then. One of the teachers had the role of an SDG leader. She was an ordinary teacher in that she taught her class various subjects, like most teachers at the school, but had voluntarily taken on the role of an SDG leader, and on that basis, she was appointed to take charge of promoting collaboration in assessing student performance and lesson planning during group meetings. Decisions about instruction, lesson plans, teaching approaches, and tasks to use were taken by the group or the individual teacher. Through training related to the leadership role, this teacher also possessed more knowledge of how to use VT in the planning and analyzing of teaching and learning than the other teachers in the group. Therefore, she also encouraged her colleagues to focus on content-specific aspects when working together on upcoming lesson plans.

Over the period of eight weeks when the teachers were planning and evaluating classroom instruction and tasks for teaching decimal numbers, the first author of this paper participated in the group. She had a double role as an observer collecting data from the meetings and a group member working closely together with the teachers. It has been pointed out that democratic values (Olin *et al.*, 2016) are a foundation for counteracting stress and dilemmas between researchers and practitioners in collaborative research projects, especially when the researcher has the role of a facilitator, providing outside support to make teacher collaboration more effective. Even though the first author in this study did not have the role of a facilitator or group leader (the group was run by the SDG leader), it was still important to achieve a democratic dialogue. Therefore, the unwritten rules on how to speak and relate to each other in the SDG (e.g. everyone in the group was encouraged to speak and different opinions and ideas were considered) were followed. For example, when the first author took an active role in the discussions, it was important to take great care not to take up more speaking space than anyone else. Moreover, even though there was a shared interest in planning and assessing teaching and learning, it was always the team's questions and

decisions that shaped the way forward in the discussions. During the weeks at the school, the first author gave a lecture on VT and its main principles for all the teachers at the school.

Mathematical content of the study

Before the study was conducted, the content in question, decimal numbers, was chosen by the teachers because it was considered difficult to teach and problematic for the students to learn. During the SDG process, the learning goal was specified as: *to develop the ability to add and multiply decimal numbers*. The challenges that students face when learning decimal numbers are well known from the literature (e.g. Durkin and Rittle-Johnson, 2015). For example, students may overgeneralize their knowledge about natural numbers when they deal with decimal numbers by assuming that a decimal number with more digits must be larger than a number with fewer decimals (e.g. they may erroneously conclude that $0.25 > 0.7$ since $25 > 7$) or that a zero can be ignored to the left of a digit in a decimal (e.g. they erroneously assume that 0.7 is equal to 0.07 since zero can be ignored in front of whole numbers, i.e. $07 = 7$).

Data and data analysis

The data consists of the eight audio-recorded group meetings for collective planning (430 min), and written observation notes supplemented with photos. Data was analyzed qualitatively using a theoretical thematic analysis (Braun and Clarke, 2006). This method is seen to be appropriate for identifying themes in studies where the research question is predetermined and contains specific theoretical interests. The analysis was carried out in the steps recommended by Braun and Clarke (2006). First, we listened to the audio-recordings and read the written observation notes several times to establish a broad view of the data, focusing on how the content was discussed in the SDG and how the discussion proceeded. In this step, we also made written notes of instances that were interpreted as teachers' use of VT principles. Based on this preliminary analysis, we formulated the questions below:

(1) In what ways do the teachers discuss students' difficulties and task solutions guided by a second-order perspective?

(2) In what ways do the teachers discuss teaching guided by variation/invariance?

Guided by these questions, instances in the recordings were selected for verbatim transcription (15 A4 pages). When reading the transcripts, we focused our attention on how the weekly discussions developed in terms of teachers' conclusions related to the two questions above. More precisely, when following the line of development of these instances in which the teachers use the principles (a second-order perspective and/or variation/invariance), we focused on teachers' conclusions about what the students need to learn and the design of student tasks, but also whether this was different from situations when the teachers did not use the principles. As the joint discussions had the nature of explorative talks (Mercer and Wegerif, 1998), in that the teachers examined their ideas from different angles or perspectives, constructively challenging each other's beliefs and presenting alternatives, this part of the analysis was not linear. On the contrary, we went back and forth in the transcripts when searching for these lines of development.

The different instances were marked with colors, grouped together, and then compared with each other. For example, instances showing *the use of a second-order perspective when evaluating student answers and what that led to* were compared with instances in which *teachers evaluated student answers by merely focusing on whether they were right or wrong*. Initially, the third author of this article analyzed the data. These preliminary findings went through a validity procedure in which they were critically scrutinized and discussed by all the authors, which yielded two revised and final themes.

Findings

The theoretical thematic analysis resulted in two themes: The principles from VT, second-order perspective and variation/invariance, contributed to (1) identification and specification of what students needed to learn and (2) design of tasks based on the identification of what students need to learn. The findings are organized under two subheadings according to the themes, presenting one example of how the teachers' assumptions and conclusions changed during collaboration and how this affected task design.

Identification and specification of what students need to learn

To identify what their students needed to learn, during the first of the weekly meetings, the teachers reflected on the challenges students face when learning about decimal numbers. The teachers drew on their professional experience and knowledge when they considered common misconceptions, skills, and the understanding that students need to develop to overcome these challenges. The collective reflections of the teachers were represented in a mind-map, which depicted what they assumed to be critical for their students' learning. Some of these assumptions were that students needed to: locate decimals on the number line, understand the place-value structure, and recognize equivalent representations for the same number (e.g. 0.7 and 0.70), which were based on the teachers' experiences of the students' failure to understand how rational numbers can be represented as fractions and as decimal numbers. Moreover, the misconceptions "longer is smaller" (e.g. $0.1504 < 0.130$) and "longer is larger" (e.g. $0.1234 > 0.180$) (Durkin and Rittle-Johnson, 2015) were also included in the mind-map. In a follow-up meeting, the teachers constructed a paper-and-pencil test on the basis of the assumed difficulties and their (probable) causes. Some of the test items were aimed at testing the students' conceptual understanding of decimal numbers (e.g. comparing pairs of decimal numbers, ordering decimal numbers with an equal number of decimals, or explaining why $0.15 > 0.105$). Other test items tested the students' computational skills, mentally (e.g. $1.4 + 0.14 =$) and with pencil and paper (e.g. $234.19 + 67.87$).

When the teachers analyzed the tests, their initial focus was on the frequency of correctly solved tasks, that is an analysis of the students' quantitative results and how these results related to what was listed in the mind-map. If a student failed to identify the larger number from a pair of decimal numbers, it was concluded that the students, for example, needed to learn how to use benchmarks (such as 0 and 0.5). At this point in the collaborative process, the participating teachers drew on their teaching experience and on a series of assumptions about what their students had to learn. However, when they reflected more deeply on their students' understanding of decimal numbers from the incorrect answers on the tests, they adopted a principle from VT, namely, the second-order perspective, in the discussion. This perspective gave rise to questions such as: "How do the students experience decimal numbers?" and "How can 1.18 as an answer make sense to them?" For example, several students failed to calculate $1.4 + 0.14$ correctly. 1.18 was a typical (erroneous) solution that was offered by the students. The teachers asked themselves why this was such a common error. They concluded that the students had failed to discern that the "4" in 1.4 and the "4" in 0.14 have different values (4 tenths vs. 4 hundredths). This insight was new to the teachers since it had not been discussed or included in the mind-map. The teachers had come to identify the different values of tenths and hundredths as a critical aspect. By adopting a second-order perspective, the teachers realized that their way of explicating what their students needed to learn had changed.

When the teachers drew conclusions about what their students needed to learn, another principle from VT guided their interpretation of their students' performance. VT states that a critical aspect must be discerned by the learner and not be merely told to the learner. From a perspective informed by VT, discernment springs from discerning variation in that aspect.

This point is reflected in the teachers' discussion about their students' conclusion that 1.4 + 0.14 = 1.18. Regarding this, one of the teachers said: "They overgeneralise their knowledge about integers". In our interpretation of her response, we claim that she explained the students' (erroneous) answer from a second-order perspective, i.e. that the students had experienced the decimal numbers as if they were integers. This insight led the topic of discussion to what the students needed to learn. One of the teachers said: "Perhaps we need to make them realize that there is a big difference between integers and decimals? And that's about learning they are not the same fractional part, isn't it?"

From the teacher's remark, we observe how she guides the discussion about what the students need to learn by raising the point that the students need to learn that there is a "big difference" between the numbers and that they should observe that "they are not the same fractional part". This remark implies a further specification of what must be learned as it relates to differences between integers and decimal numbers that the students must learn. Or more succinctly, the students must learn to distinguish position values in integers from position values in a decimal number.

Design of tasks based on the identification of what students need to learn

How such a critical aspect could be included in the next lesson was discussed by the teachers and made manifest in the design of the learning tasks that were given to the students. With respect to identifying the critical aspect (i.e. to distinguish position values in integers from position values in a decimal number), one teacher suggested juxtaposing and comparing pairs of numbers in the lesson, saying:

> Let's start with 7 and 35 [on the board] and ask which is the bigger? And next, we can take 0.7 and 0.35 and ask: But what about these? We start with a contrast and look at the integers and compare them. What's the difference? Because here [35 and 7] they all know that 35 is bigger than 7, but here [0.35 and 0.7] 35 is not bigger anymore.

The suggestion led to the following task:

(1) Compare 35 and 7. Which is the larger number?

(2) Compare 0.35 and 0.7.

(3) What is the difference between no. 2 and no. 1?

In the above task, we can see that principles from VT were used in the design. First, the decision to compare a pair of numbers by juxtaposing the numbers and making a contrast resonates with the VT principle that seeing how things differ takes precedence over seeing sameness. Second, we notice that the same numerals, 35 and 7, were used in both examples. In the first example, the numerals are presented as integers, while in the second example, they are presented as decimal numbers. This implied a systematic pattern of variation: the numerals remained invariant, whereas their position, and hence their value, varied. This example also followed two principles from VT, namely (1) that which varies will most likely be discerned and (2) discernment of a critical aspect is only made possible by opening it as a dimension of variation.

Conclusion and discussion

It has been described how VT can contribute to teachers developing knowledge about what students need to learn and how teaching can be designed to provide learning opportunities (e.g. Holmqvist, 2011; Lo *et al.*, 2005; Pang and Lo, 2012). However, since VT has been strongly linked to LearS, we wanted to investigate whether and how it is useful in other forms of collaborative work. Therefore, we studied a group of teachers collaborating in an

arrangement—SDG—that differs from LearS in several respects. Our research question addressed how principles from VT; a second-order perspective and variation/invariance, contribute to teachers' planning of teaching and learning when participating in SDG. The results show that the two principles were used to assist the teachers in identifying and specifying what was critical for their students' learning. Furthermore, these principles guided the teachers when they planned and designed mathematics tasks. Although the current study deviated from a traditional LearS, our results support previous research on the efficacy of VT in LearS (e.g. Holmqvist, 2011; Pang and Lo, 2012; Ting *et al.*, 2018) and indicate that VT might also be beneficial in other forms of collaborative work beyond LearS.

As this study examines teachers' collaborative work using a model that does not involve the observation and repeated revision of a single lesson which characterizes LearS, it is tempting to question whether or to what extent these are necessary for the planning and assessment of teaching and learning. The benefits of getting information from sources such as video recordings or lesson observations, in that the common experience of one single lesson can be discussed, must be considered as valuable in the process. However, if, for various reasons, is not possible to implement LearS at a school, this study shows that the SDGs can serve the same purpose.

It could be argued that teachers could have come to the same insights without being guided by VT. We claim, however, that the study demonstrates instances where the teachers' existing knowledge about student learning was changed by adopting principles from VT. Our results show that the participating teachers started by explicating their experience of teaching and learning a specific mathematical topic, and the assumed critical aspects (relevant to the teaching and learning of this topic) were described in terms of a list of aspects related to the discipline. We interpret these reflections as necessary to initiate the identification of what is critical for student learning. However, when the teachers abandoned their typical analysis of the students' results on the pre-test (i.e. the number of correct answers) and adopted a principle from VT (a second-order perspective) as a theoretical "lens" to examine and understand the students' reasoning in their incorrect answers, they gained new insights into what the students needed to distinguish. We argue that this seemingly subtle difference is a matter of great importance, because "distinguishing" (as the term is used in VT) not only highlights what students must learn but also captures what this means exactly and how this must be taught. The identified critical aspect indicates that which must be juxtaposed and compared to make differences discernible. These insights were further developed and made more specific regarding how the teachers could promote students' learning when the principle of variation/invariance informed the planning of the tasks. The task presented above clearly reflects principles from VT: how numbers in the task remain invariant whereas the positions vary. Consequently, we observed how principles from VT gave the teachers a new direction for their classroom instruction, namely, how to handle the specific lesson content and how to assist their students in discerning the critical aspects relevant to learning about addition of decimal numbers.

Finally, we present some comments on the limitations of our study. One is that we only studied the teachers' ways of reasoning during their collective reflections on teaching and learning about decimals and how they transformed their experience of teaching and learning decimals into new knowledge. We did not examine how their subsequent teaching plans were implemented. Neither did we study student learning, i.e. whether (and in what way) the students' ability to add decimals was improved, and hence we cannot say anything about the effects of SDG on student learning, similarly to studies on LearS (e.g. Kullberg *et al.*, 2016b).

The role of the SDG leader also needs to be discussed. She was appointed and had the formal role of leading the process and was more familiar with VT than the other teachers. It has been reported that an outside expert—a knowledgeable other—makes the lesson study more effective (e.g. Takahashi, 2014). The SDG leader was an insider, however, and although

she may not have had the same authority as an outside expert, her role of promoting new ideas and different perspectives was probably important for the development of the process. One of the authors participating in the group meetings must also be considered, as she was more of a knowledgeable other, an outside expert. Being aware of the impact she might have on the discussion and decisions, she tried to adhere to the democratic values (Olin *et al.*, 2016) previously described, for example, by staying in the background as much as possible, to let the SDG leader have the leading role. However, it is not possible to say what effect she might have had on the interaction in the group. This could be further explored in future studies within as well beyond LearS.

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