Editorial: Comment on "MHD-mixed convection flow in a lid-driven trapezoidal cavity under uniformly/non-uniformly heated bottom wall"

1. Comments

Javed *et al.* (2017) performed a numerical investigation of MHD-mixed convection heat transfer inside a lid-driven trapezoidal cavity with uniform or non-uniform heating conditions. During their simulation, they varied the governing dimensionless parameters such as Rayleigh number (Ra) and Prandtl number (Pr). For a mixed convective problem, the associated heat and fluid flow characteristics due to the combined effect of natural and forced convection mechanisms are mathematically described by the combination of two crucial governing parameters, namely, Grashof number ($Gr = Ra/Pr$, which signifies the dominance of natural convection) and Reynolds number (Re, which indicates the dominance of forced convection). Hence, the characteristic parameter for mixed convection heat transfer is called the Richardson number ($Ri = Gr/Re^2$), which should be $0.1 \leq Ri \leq 10$ within the mixed convection regime (Lukose and Basak, 2021). Forced convection becomes dominant when $Ri > 0.1$, while the dominance of natural convection takes into play at $Ri > 10$. When $Ri = 1$, it is commonly known as pure mixed convection (Rahman *et al.*, 2010; Hasib *et al.*, 2015). Unfortunately, Javed *et al.* (2017) did not consider either Richardson or Reynolds number in their investigation while performing an extensive analysis of the mixed convection problem. They also mentioned that they varied the Hartmann number within $50 \leq Ha \leq 1{,}000$ to show the MHD effect but presented all results only at $Ha = 50$. Besides, they mentioned the investigated range of the Prandtl number twice in their paper as $0.026 \leq Pr \leq 100$ and $0.026 \leq Pr \leq 1{,}000$, respectively, but only considered $Pr = 0.026$ and 10.

Javed *et al.* (2017) considered a lid-driven trapezoidal cavity in their problem, where the top lid was moving at a fixed speed U_0. Hence, they mentioned the velocity boundary condition of the top wall as u(x, L, where L is the height of the trapezoidal enclosure) $= U_0$ in equation (5) of their paper, which represents the sliding motion of the lid in the positive x-direction. However, while drawing the schematic diagram of their problem [see Figure 1(a)], they showed the direction of the lid movement in the negative x-direction, which was wrong as per the boundary condition described in equation (5). Moreover, they neither showed the size of the base wall of the trapezoidal chamber in Figure 1(a) nor mentioned it anywhere in their paper. By analyzing Figure 9 of Javed *et al.* (2017), it can be confirmed that the base length of the trapezoidal cavity was L. The corrected physical domain is now shown in Figure 1(b), along with the corrected direction of the lid movement. In the caption of Figure 1 of Javed *et al.* (2017), it was mentioned that $\phi = 0°$ represented the square cavity, where ϕ was the inclination angle of the side walls. However, the physical models developed using the physical dimensions of three different cases ($\phi = 0°$, 30° and 45°) do not match with the domain shown in various qualitative plots (Figures

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3–8) of Javed *et al.* (2017). A comparison of the difference between these physical models is presented in Figure 2. Besides, Javed *et al.* (2017) incorrectly mentioned the profiles of inclined side walls of the trapezoidal cavity in equation (5) as $x\sin\phi + y\cos\phi = 0$ (left side wall) and $x\sin\phi - y\cos\phi = L\cos\phi$ (right side wall). It should be correctly expressed as $x\cos\phi + y\sin\phi = 0$ and $x\cos\phi - y\sin\phi = L\sin\phi$, respectively.

Javed *et al.* (2017) used α/L as the reference velocity and $\rho\alpha^2/L^2$ as the reference pressure in equation (6) to obtain the non-dimensional governing equations [equations (7)–(10)] of their problem. These reference velocity and pressure are usually used for natural convection problems. Hence, the non-dimensional mathematical model mentioned in equations (7)–(10) of their paper is commonly used for analyzing natural convection problems. The appropriate reference velocity and pressure of their problem should be U_0 and ρU_0^2, respectively, which result in the corrected form of the non-dimensional mathematical model applicable for analyzing mixed convection problems. Javed *et al.* (2017) reviewed two works related to mixed convection (Moallemi and Jang, 1992; Sheremet and Pop, 2015) but did not follow the non-dimensional mathematical model mentioned in those papers. Javed *et al.* (2017) also wrote the incorrect non-dimensional continuity equation (7) in their paper. The corrected dimensionless continuity equation is as follows:

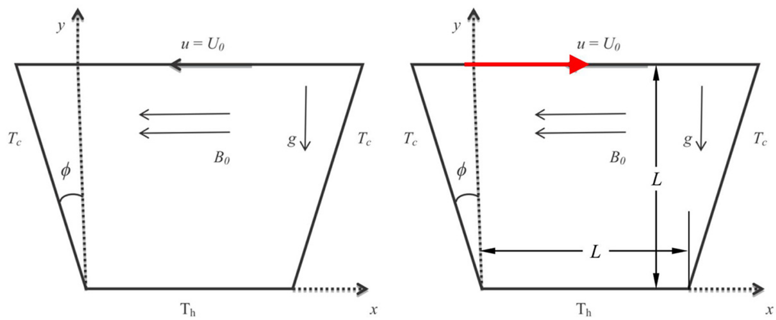

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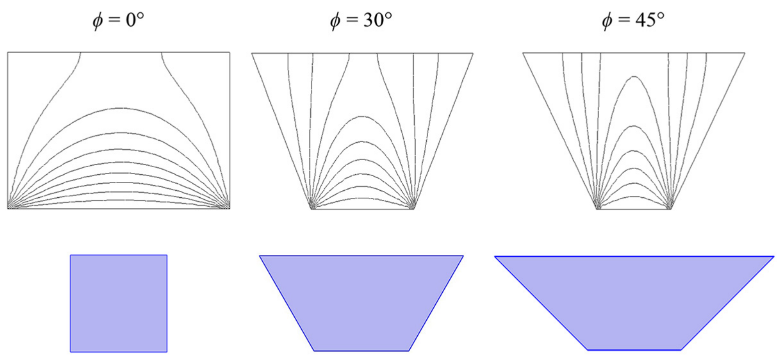
Figure 1. (a) Schematic illustration of the physical model considered by Javed *et al.* (2017), and (b) the corrected physical model of the problem by Javed *et al.* (2017) showing the corrected direction of wall movement (color online)



Source: Figure courtesy of Javed *et al.* (2017)

Figure 2. Comparison of the physical model between Javed *et al.* (2017) and the present analysis (obtained from the corrected dimensions) for various inclination angles of the side walls (color online)

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0. \tag{1}$$

Using the correct reference velocity and pressure, the dimensionless momentum and heat energy equations for the MHD-mixed convection problem of Javed *et al.* (2017) can be written as follows:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\theta-\frac{Ha^2}{Re}V, \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right). \tag{4}$$

where the Reynolds number $Re = U_0L/\nu$ was not defined in the paper of Javed *et al.* (2017).

With the reference velocity considered by Javed *et al.* (2017), the corresponding non-dimensional velocity boundary condition at the top wall becomes $U(X, 1) = U_0L/\alpha = RePr$. However, they mentioned $U(X, 1) = 1$ in equation (11) of their paper. With this setting of velocity boundary condition $U(X, 1) = RePr = 1$, the unknown governing parameter, Reynolds number can be calculated as $Re = 1/Pr$. Since Javed *et al.* (2017) considered $Pr = 0.026$, $Pr = 10$ and $10^3 \leq Ra \leq 10^6$ in their simulation, the corresponding values of Richardson number ($Ri = Ra/PrRe^2$) can be evaluated as listed in Table 1. It is found from Table 1 that the range of Richardson number considered in the problem of Javed *et al.* (2017) is much higher than the usual range of mixed convection regime ($0.1 \leq Ri \leq 10$). Hence, it can be concluded that Javed *et al.* (2017) did not analyze mixed convection phenomena in their paper. Instead, they examined the natural convective flow inside the cavity.

The non-dimensional thermal boundary conditions mentioned in equation (11) by Javed *et al.* (2017) were incorrectly expressed using the dimensional temperature variable. It should be corrected as follows:

- bottom wall: $\theta(X, 0) = 1$ or $\sin \pi X$;
- top wall: $\partial\theta(X, 1)/\partial Y = 0$;
- left side wall: $\theta(X, Y) = 0$ at $X\cos\phi + Y\sin\phi = 0$ and $0 \leq Y \leq 1$; and
- right side wall: $\theta(X, Y) = 0$ at $X\cos\phi - Y\sin\phi = \sin\phi$ and $0 \leq Y \leq 1$.

The figure captions of Figures 5 and 8 in the paper of Javed *et al.* (2017) incorrectly displayed the value of the Prandtl number as $Pr = 0.026$. It should be $Pr = 10$, which was clearly mentioned in the *Results and Discussion* section of Javed *et al.* (2017).

Table 1. Values of different governing parameters considered in the analysis of Javed *et al.* (2017)

Pr	*Re*	*Ra*	*Ri*	Comment
0.026	38.46	$10^3 \leq Ra \leq 10^6$	$26 \leq Ri \leq 2.6 \times 10^4$	Natural convection regime ($Ri > 10$)
10	0.1	$10^3 \leq Ra \leq 10^6$	$10^4 \leq Ri \leq 10^7$	Natural convection regime ($Ri > 10$)

Source: Table by authors

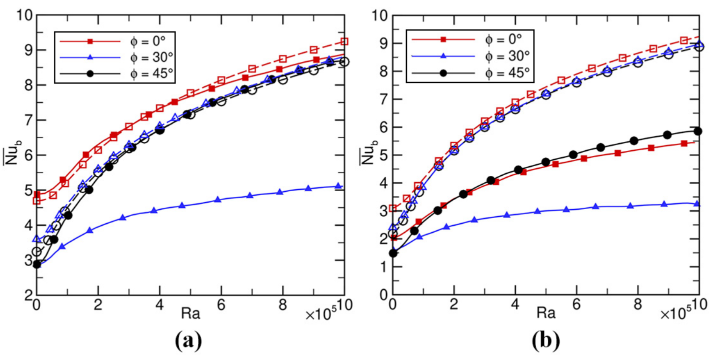
Javed *et al.* (2017) considered uniform and non-uniform heating conditions for three different inclination angles of the side walls of the trapezoidal cavity to evaluate the mean Nusselt number along the heated bottom wall, presented in Figure 10 of their paper. However, they did not mention the formula for calculating the mean Nusselt number at the heated bottom wall. For the uniform isothermal heating condition, the mean Nusselt number ($\overline{Nu_b}$) at the bottom wall is expressed as follows:

$$\overline{Nu_b} = -\int_0^1 \frac{\partial\theta}{\partial Y}\bigg|_{Y=0} dX. \tag{5}$$

On the other hand, for a non-uniform (sinusoidally) heated surface, the correct expression of the mean Nusselt number along the bottom wall can be expressed following the derivation of the recent publications (Shuvo *et al.*, 2023; Deb and Saha, 2024) as follows:

$$\overline{Nu_b} = -\frac{\int_0^1 \frac{\partial\theta}{\partial Y}\Big|_{Y=0} dX}{\int_0^1 \theta\Big|_{Y=0} dX} = -\frac{\int_0^1 \frac{\partial\theta}{\partial Y}\Big|_{Y=0} dX}{\int_0^1 \sin(\pi X)dX} = -\frac{\pi}{2}\int_0^1 \frac{\partial\theta}{\partial Y}\bigg|_{Y=0} dX. \tag{6}$$

Unfortunately, in their paper, Javed *et al.* (2017) did not use the correct expression of the mean Nusselt number at the heated bottom surface. Hence, a correction in their results is required, as illustrated in Figure 3. The data from the present simulation is generated using a Galerkin finite element method-based simulation CFD software "COMSOL Multiphysics 6.2". It is clearly observed that the results of Javed *et al.* (2017) and the current simulation match quite well for the uniform heating condition when $\phi = 0°$ and $45°$ following the correct formulation of equation (5). However, when $\phi = 30°$, the data from Javed *et al.* (2017) underpredict the value of mean Nusselt number compared to the present simulation case. It



Notes: (a) Uniform heating and (b) non-uniform heating. The solid and dashed lines show the data for Javed *et al.* (2017) and the present simulation, respectively (color online)
Source: Figure by authors

Figure 3. Comparison of the mean Nusselt number computed using the correct formulation with those of Javed *et al.* (2017) at $Pr = 0.026$, $Ha = 50$

is unacceptable to have such a variation of the mean Nusselt number for the case of $\phi = 30°$ without any dramatic change in flow and thermal fields, which Javed *et al.* (2017) failed to explain clearly. Hence, it can be undoubtedly confirmed that Javed *et al.* (2017) computed the incorrect mean Nusselt numbers for the case of $\phi = 30°$. Moreover, the case of non-uniform (sinusoidal) heating shows a significant deviation between the present computation [based on the correct formulation of equation (6)] and the data given in Javed *et al.* (2017). The pattern of the mean Nusselt number has an inconsistent trend with the increase of the inclination angle of the side walls. In contrast, the present simulation displays a consistent alteration of the Nusselt number with Ra for all magnitudes of ϕ. Because Javed *et al.* (2017) did not mention any formula for the mean Nusselt number, it can be confirmed that the deviation of these two sets of data for non-uniform heating is due to the usage of the wrong formulation of Nusselt number calculation by Javed *et al.* (2017).

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