Market competition, labor value and price: a Marxist disequilibrium theoretical and empirical framework

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Abstract

Purpose – The correlations and deviations between market prices, production prices and values are critical indexes for testing the labor theory of value. However, there is not yet a universally accepted method of solving for the production price vector, and given the complexity and volatility of economic dynamics, production prices based on the assumption of economic equilibrium cannot be accurate measuring. This paper attempts to propose a new approach to test the labor theory of value.

Design/methodology/approach – This paper proposes a different approach. From the perspective of disequilibrium price, the paper deduces that the range of the relative prices of commodities is determined by the relative value, the rate of surplus-value and the technical structure of production inputs, with relative price fluctuating within the value range specified by the labor theory of value under market competition influences.

Findings – With empirical research results based on China's economic data, this paper not only affirms the scientific and practical explanatory power of the labor theory of value in a more general sense but also uncovers how surplus value is distributed across sectors, which can be used to analyze market competition and technical relations and their impacts on industrial structure and distribution.

Originality/value – The disequilibrium analytical framework provides a new perspective for the empirical study of labor theory of value. Moreover, it evolves the labor theory of value into a robust empirical framework, breaking through the theoretical path of the traditional labor theory of value that is mostly limited to the normative discussion of exploitation.

Keywords Disequilibrium, Labor theory of value, Relative price, Relative value

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1. Introduction

The labor theory of value is the cornerstone of scientific Marxist economics and remains a highly controversial area within both Marxist economists and other economic schools. One of the most famous and enduring controversies lies in the so-called "transformation problem". In particular, after World War II, theoretical questions on this topic were introduced from German academic circles to English-speaking scholars, notably through the work of Paul Sweezy. This infusion of ideas sparked extensive and systematic research by Marxist economists, aimed at defending the labor theory of value against theoretical criticisms, as well as refining and further developing Marx's original concepts regarding the labor theory of value. Relevant studies mainly follow the following two research approaches.

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The first is fundamentally theoretical, demonstrating the logical coherence of the transformation from value to production price through rigorous theoretical reasoning. The standard solution developed in accordance with Bortkiewicz's analytical tradition facilitates a shift toward standardized mathematical expressions in the study of value transformation. Since the 1970s, significant theoretical advancements have been achieved in Marxist economics. The "New Interpretation" proposed by Foley and Duménil addresses the issue of equal aggregate values before and after the transformation process. Chinese scholars Rong Zhaozi and Chen Yang approached the transformation problem by upholding the invariant condition that the aggregate value of commodities equals the aggregate production price and the overall rate of surplus-value remains unchanged, revealing the logical mechanism inherent in consistent distribution relation before and after the transformation (Chen and Rong, 2018; Rong and Chen, 2014, 2022). These studies reveal the intrinsic theoretical logic underlying the transformation of value into production price from various theoretical perspectives, enhancing the validity of the labor theory of value through theoretical consistency.

The second research approach is empirical or evidence-based. It aims to empirically demonstrate that production prices act as the center of gravity for market price fluctuations, while values form the basis for the formation of production prices. This approach is intended to argue for the practical explanatory power of the labor theory of value. Such studies employ input-output data to measure the correlations and deviations between market prices, production prices and values across various countries and regions to prove that fluctuations in both market price and production price are fundamentally influenced or determined by value. This empirical validation aligns the labor theory of value with tangible evidence from reality, advancing empirical studies within Marxist political economy. However, the existing empirical methods of studying the labor theory of value exhibit technical and theoretical limitations. First, the limitations in the measurement indexes and the controversy over the solution of the production price vector lead to discrepancies in the calculation results between different models and economic data from various economies, making it challenging to establish a unified measurement standard. More critically, production price is an abstract category formed under the condition of complete market competition equilibrium. Economic activities in the real world are characterized by complexity and volatility; thus, theoretically, derived production prices based on the premise of complete competition equilibrium do not directly correspond to market prices. This theoretical limitation results in contradictions between theory and reality in empirical studies [1].

Based on previous research, this paper shifts from arguing the correlation or deviation between market prices, production prices and values to arguing that the range of market prices can be explained by the labor theory of value. This study has two goals: To reveal the practical explanatory power of the labor theory of value in a more general sense while seeking to develop the labor theory of value into an empirical research framework for examining real economic issues. To this end, we prove through a disequilibrium value-price model that the range of relative prices is determined by relative values, the rate of surplus-value and the technical structure of production inputs, while the relative prices fluctuate within the range determined by the labor theory of value under the influence of market competition. Traditional methods of testing the labor theory of value focus on measuring the relationships between market prices, production prices and values, which can be seen as a form of "point estimation". In contrast, our approach focuses on demonstrating that the range of prices is determined by categories relevant to the labor theory of value, which can be viewed as a form of "interval estimation". The advantage of this "interval estimation" approach is that it avoids the discrepancies in point estimation results caused by factors such as price volatility, the selection of indexes and the solution of production prices, which may weaken the interpretive power of empirical findings. Consequently, it enables a more general validation of the empirical explanatory power of the labor theory of value, which offers a new perspective for the empirical studies on the labor theory of value.

The scientific validity of a theory is demonstrated not only by its internal logical consistency but also by its practical applicability. However, traditional theoretical and empirical studies on the labor theory of value focus more on demonstrating the consistency of the theory and empirical logic or discussing labor "exploitation" with less attention to the theory's implications for economic practices. This paper introduces an analytical framework that can examine the flow of value across various sectors under real market competition, i.e. the redistribution of value across various sectors of society. Based on this, the labor theory of value can be developed into an empirical research framework for examining how factors such as market competition and technical conditions affect the industrial structure and distribution relations, striving to go beyond the traditional labor theory of value limited to normative discussions of issues like exploitation. Based on this, by incorporating specific economic factors such as market competition forces and the transformation of market systems, the labor theory of value can be further developed into a problem-oriented theoretical and empirical research framework.

The remainder of this paper is organized as follows: Section 2 critically reviews and evaluates the common methods employed in existing empirical studies on the labor theory of value; Section 3 presents the value-price disequilibrium model proposed herein; Section 4 conducts an empirical analysis utilizing data on the Chinese economy, while the concluding section synthesizes the findings and implications.

2. Literature review: common methods and evaluations for empirical studies on the labor theory of value

This section systematically summarizes the methods for the empirical studies on the labor theory of value and analyzes the divergences and controversies among existing approaches. The reasons for the difficulty in resolving these disputes and disagreements lie in the differences between various theoretical models under equilibrium conditions and the fact that existing research predominantly applies equilibrium models without considering the disequilibrium and volatility in empirical data. Therefore, by comparing and reviewing existing research, this paper aims to more comprehensively demonstrate the theoretical significance and practical value of the method employed herein.

2.1 The transformation of value into production price: a general model

A fundamental issue in the debate over the labor theory of value is how to transform the abstract value of a commodity into a measurable entity. Morishima (1973) proposed a method to measure value, revolutionizing value theory by bridging the gap between pure theoretical discussion and empirical research. Based on the input–output relationship of commodity production, value can be mathematically expressed as follows:

$$t_i = a_{i1}t_1 + \cdots a_{in}t_n + \tau_i \tag{1}$$

where t_i is the value per unit product produced by sector i, including the transferred value of intermediate inputs (embodied labor) and the newly added value (living labor), a_{ij} represents the amount of intermediate input from sector j consumed by sector i to produce one unit product and τ_i is the direct labor input required to produce one unit product by sector i.

Assuming there are n production sectors, let t represent the value vector, A the matrix of direct consumption coefficients and τ the labor input vector. Equation (1) can be expressed in the following matrix form:

$$t = At + \tau \tag{2}$$

The value-row vector can be solved in the physical input–output table as follows:

$$t = (I - A)^{-1}\tau \tag{3}$$

Equation (3) can only be applied to physical input–output tables. However, most countries and regions compiled value-based input–output tables. Thus, it is necessary to transform Equation (3) into a form that can be applied to value-based input–output tables:

$$\hat{t} = \left(I - \hat{A}\right)^{-1}\hat{\tau} \tag{4}$$

where $\hat{t}_i = \frac{t_i}{p_i}$, $\hat{a}^*_{ij} = a_{ij}\frac{p_j}{p_i}$, $\hat{\tau}^*_i = \frac{\tau_i}{p_i}$ and p_i is the market price of one unit product produced by sector i. At this point, as long as we get the column vector of direct labor input and the direct consumption coefficient matrix, we can calculate the labor values of commodities with unit monetary value in the value-based input–output table.

The estimation of production price falls within the realm of the transformation problem in Marxist political economy. According to Marx's definition [2], the production price column vector can be defined as follows:

$$p = (1 + r)(Ap + w\tau) \tag{5}$$

In Equation (5), w denotes the nominal wage, and r the average profit rate. The system of equations comprises $n+2$ unknowns, yet only n linear equations. The solution of p and r requires two additional constraints, denoting the transformation problem of Marxist political economy, which has been studied by Rong and Chen (2014, 2018, 2022) systematically. According to the various supplementary constraints, the solution of the transformation problem is divided into Systems A, B and C as follows:

$$\text{System A}\begin{cases} p = (1 + r)(Ap + w\tau) \\ w = b^T p \\ p^T x = t^T x \end{cases} \tag{6}$$

$$\text{System B}\begin{cases} p = (1 + r)(Ap + w\tau) \\ r(Ap + w\tau)^T x = e\omega\tau^T x \\ p^T x = t^T x \end{cases} \tag{7}$$

$$\text{System C}\begin{cases} p = (1 + r)(Ap + w\tau) \\ \dfrac{p^T y}{t^T y} = \dfrac{w}{\omega} \\ p^T x = t^T x \end{cases} \tag{8}$$

where e is the rate of surplus-value which is the rate measured by value, x thee column vector of total output and y the net output-wage rate. The equation in the third row of the three systems is the same; namely, the total price equals the total value. The difference lies in the second condition: System A assumes that workers receive given wages in kind, System B assumes that total profits are equal to total value and System C assumes that the aggregate rate of surplus-value remains unchanged [3]. Using a method similar to Equation (4), we can find p^*, the column vector of the production price of a commodity with unit monetary value in the value-based input–output table.

2.2 Empirical test and application of the relationships between market price, value and production price: mainstream methods and evaluations

Since the 1980s, researchers such as Shaikh (1984) and Ochoa (1989) have initiated empirical investigations into the labor theory of value by calculating production prices and values derived from input–output tables. By computing these production prices and values, one can derive the labor value vector t^* and the production price vector p^* of a unit currency value of the commodity, as well as the market price vector 1 of the commodities (in a value-based input-output table, this market price vector is characterized as a column vector with all elements equal to 1). The dimension of labor value is measured in terms of labor time, while the production price vector [4] and market price are quantified in monetary units. Therefore, direct comparisons among these three vectors are not feasible. Therefore, it becomes essential to convert the labor value vector t^* of the commodity into its monetary equivalent, Λ, as follows:

$$\Lambda = \hat{t}\left(\frac{1^T x}{\hat{t}^T x}\right) \tag{9}$$

At this point, we can derive dimensionally consistent vectors of commodity value, production price and market price. We can assess whether empirical data supports the labor theory of value by measuring the relationships between these three vectors. The methods for evaluating these relationships can be categorized into two types: comparing correlation coefficients or conducting regression analyses between each pair of vectors and quantifying the degree of deviation between each pair. If the calculated vectors demonstrate high correlation coefficients, regression coefficients approaching unity or minimal deviations, the explanatory power of the labor theory of value will be proved by empirical evidence.

Utilizing input–output table data from Italy and the USA, Shaikh (1984) quantified prices and values, uncovering a correlation exceeding 90% between the two. This finding underscores the pivotal role of labor value in shaping market prices. Building upon Shaikh's (1984) methodological framework, subsequent studies have further advanced empirical investigations into the labor theory of value by broadening data samples and refining measurement indexes. Empirical investigations by Petrović (1987), Ochoa (1989) and Cockshott *et al.* (1995), utilizing input–output table data from the USA, former Yugoslavia and the United Kingdom, consistently demonstrate that deviations between market prices, production prices and values generally remain within 15%, which substantiates the pivotal role of value in influencing both production price and market price. Research by Chinese scholars (Li, 2017; Ma, 2018; Ren and Wang, 2023) systematically expounded the characteristics of relevant indexes for measuring the relationships between market price, value and production price. Based on the data from China's input–output table, they measured the correlation and deviation between market prices, values and production prices under different measurement indexes and obtained similar empirical results, providing more robust data to support the conclusions of existing empirical research. Cheng and Li (2020) utilized data from China's input–output tables to examine the relationships between market price, value, Marxian production price and Sraffian production price, whose research results indicated that the deviations between all these indexes are relatively minor and that the deviations of both Marxian and Sraffian production prices from market price are less than that of value from market price, suggesting that production price instead of value is the center of gravity for market fluctuations. Işıkara and Mokre (2022) broadened the scope of their investigation to encompass input-output data from 42 countries over 15 years. Their findings confirmed that the deviation of value from production price is smaller than that of value from market price. The empirical results obtained across various nations and regions substantiate the real-world explanatory power of the labor theory of value, illustrating its capacity to integrate theoretical frameworks and practical applications effectively.

The above methods for testing the labor theory of value demonstrate the theoretical and empirical logical coherence of the theory, thereby affirming the scientific validity of the theory and more importantly, enable the measurement of such essential variables of political economy as labor value and production price, advancing the empirical study in the field of political economy. Some scholars have undertaken empirical investigations into China's economic growth, structural dynamics, income distribution and technical advancements based on Marx's two-sector growth model or its expanded three-sector variant (Wang and Liu, 2018; Li *et al*., 2019; Xu and Liu, 2022). Other researchers have applied economic circulation theory to examine regional disparities and patterns of economic circulation (Feng *et al*., 2020; Li *et al*., 2021; Qiao *et al*., 2023). Furthermore, some investigators have adapted reproduction models to encompass the global reproduction system to empirically analyze the formation of international value and international production price (Feng, 2016; Liu and Song, 2017). This type of literature either directly employs the input–output method in empirical research to calculate such variables as labor value and production price for exploring actual economic issues or uses market price as a proxy variable for labor value or production price in empirical analyses, which implicitly assumes that, in the long run, the deviations between values, production prices and market prices are small – a hypothesis supported by numerous empirical studies examining the labor theory of value.

However, as research in this field advanced, there have been criticisms on the empirical approach of employing input–output analysis to evaluate the labor theory of value from both technical and theoretical perspectives. Thus, the validity of related empirical studies in political economy that emerged from this methodological framework necessitates further explanation.

Firstly, on the technical level, there are certain methodological limitations in measuring the correlations, regression coefficients and deviations between values, production prices and market prices. As for the correlation and regression coefficients, since the market prices in the value-based input-output table consist of a vector of all ones, the correlation coefficient cannot be mathematically defined. Additionally, the coefficients obtained from regressing market prices represent the expectations of commodity values and prices only. To address this limitation, the academic community has begun to shift toward measuring the correlation or regression coefficients between the aggregate market prices, aggregate values and aggregate production prices of various sectors. However, it is crucial to note that such correlations primarily arise from the correlations between aggregate outputs rather than market prices, value and production costs. Thus, these findings are deemed unreliable (Kliman, 2002). Consequently, researchers have regarded deviations between vectors of market price, value and production price as indexes to test the practical explanatory power of the labor theory of value (Shaikh, 2016). The deviation indexes can be classified into two primary categories. The first category encompasses deviation indexes with units of measurement, such as mean absolute deviation (MAD), mean absolute weighted deviation (MAWD) and normalized vector distance (NVD). The values of these indexes are linked to the units of measurement associated with the calculated vectors, rendering comparisons between deviation indicators over different years impractical. This limitation poses a significant challenge for evaluating the practical explanatory power of the labor theory of value. The second category consists of deviation indicators without units of measurement, which primarily assess deviations based on the angles between vectors, effectively overcoming the limitations inherent in the first category and allowing for comparative analyses of deviations across different years. They are regarded as relatively robust and reliable among current indexes employed to test the labor theory of value. However, there are also limitations associated with the angle-based deviation indexes. When the deviations of market prices from their corresponding values or production prices vary across different sectors, the angles between vectors may remain unchanged. In other words, this index may not adequately reflect the structural differences in deviations. In addition, since all three vectors are derived from the same value-based input-output table, the high correlations, regression coefficients approaching 1 and small deviations observed

between market prices, values and production prices may result from intrinsic correlations generated by linear calculation methods rather than from theoretical cause-and-effect relationships.

Secondly, from the theoretical perspective, the existing methodologies for testing the labor theory of value necessitate further refinement. A key step in existing empirical studies testing the labor theory of value is the estimation of production prices. However, the estimation of production prices involves multiple methods, each corresponding to different assumptions, resulting in varying production prices. Consequently, there are multiple possible outcomes when measuring the relationship between market prices, production prices and values, and no universally accepted standard exists. Researchers must choose interpretative frameworks in practical research based on their specific needs. Therefore, the validity of testing the labor theory of value remains open to further discussion. More importantly, in Marx's theoretical framework, the formation of production prices results from the equalization of profit rates. This equalization is a theoretical law discovered by Marx as he moved from concrete analyses to higher levels of abstraction, logically revealing changes in how the law of value operates. However, abstraction and concreteness do not have a one-to-one correspondence. Applying abstract categories directly to concrete issues contravenes the requirements of the scientific method of abstraction. In empirical research, production prices calculated based on the average profit rate can only be considered a special case in the value realization process (Zambelli, 2018). Marx (2004, p. 181) also points out that as for the production price, "in the whole capitalist production, the general law, as a dominant trend, always works only in an extremely complicated and approximate way, as a constantly fluctuating but never certain mean value." The equalization of profit rates is achieved through a constantly complex and fluctuating process. Since various factors obstruct the process of equalizing profit rates, the equalization of profit rates can only be perceived as a long-term trend. Therefore, in theory, although production prices adjust the capitalist relationships as a transformed category of value, the volatility and complexity of economic dynamics make it difficult to accurately measure the specific values of production prices empirically. This theoretical limitation has also been reflected in some empirical studies, where the deviation of market price from production price is greater than the deviation of market price from value (Ochoa, 1989; Tsoulfidis and Maniatis, 2002). However, according to Marx, the transformation of value into production price is a transformation process from abstraction to concreteness, and the price of production, instead of value, becomes the center of gravity for market price fluctuations, so the deviation of market price from production price should be smaller than the deviation of market price from value logically. In the empirical research literature on Marxian economics, some parts deal with the calculation of production prices. The above issues can impact the validity of the empirical research findings based on such calculations to a certain extent. Moreover, the production prices as a result of the equalization of profit rates represent a general rule in capitalist economies and often serve as a reference standard. Therefore, whether from the perspective of testing the labor theory of value or from the standpoint of refining empirical research in political economy, we need to improve the methods for estimating production prices or develop new variables to substitute the reference standard function of production price.

It is worth mentioning that Farjoun and Machover (1983) and Schefold (2013, 2019) introduced probability theory and quantitative statistics into the empirical study of political economy. They view the capitalist economy as a complex and turbulent process where profit rates, prices, production technologies, etc. follow a certain random probability distribution. Based on this framework, they studied issues related to the labor theory of value. This aligns with the "disequilibrium" perspective emphasized in this paper. The approach of solving for market prices and production prices under uncertain conditions can be considered theoretically sound. However, their assumption about specific probability distributions for profit rates and prices lacks a corresponding theoretical basis. The closeness of their model to real-world data on profit rates and prices might be a result of data fitting. Furthermore, their conclusions have

not been strongly supported by empirical tests of data from some economies (Basu, 2017; Mohun and Veneziani, 2017). Therefore, this paper abandons the theoretical assumption of specific probability distributions but draws upon the research approach of interpreting the labor theory of value under conditions of uncertainty.

Herein, we put forward a new approach to test the labor theory of value, shifting from arguing the correlations or deviations between market price, production price and value to demonstrating that the range of market prices can be explained by the labor theory of value, thereby revealing the practical explanatory power of the labor theory of value in a more general sense. To this end, we construct a disequilibrium-price model to prove that the range of the relative market price that a commodity can finally achieve can be explained by the labor theory of value in the third section. The model results show that the range of the market prices of commodities is determined by three factors: the relative value of the commodity, the rate of surplus-value and the technical conditions for production.

3. The disequilibrium value–price model

In this section, we construct a value-price disequilibrium model based on extending the theoretical model of Nakatani and Okishio (1995). We argue the relationship between relative prices and relative values from the perspective of disequilibrium prices.

In an *n*-sector model, the constraints for capitalists in each sector to obtain positive profits [5] are as follows:

$$p_i > \sum_{j=1}^{n} a_{ij} p_j + \tau_i w \tag{10}$$

$$w = \sum_{j=1}^{n} b_j p_j \tag{11}$$

where p_i is the market price of one unit product produced by sector i, a_{ij} is the intermediate input of sector j consumed by sector i to produce one unit product, τ_i is the direct labor input of sector i to produce one unit product and $b = (b_1, \cdots, b_n)^T$ the vector of in-kind wage. The labor value t_i and the rate of surplus-value e per unit commodity can be expressed as follows:

$$t_i = \sum_{j=1}^{n} a_{ij} t_j + \tau_i \tag{12}$$

$$e = \frac{1 - \sum_{j=1}^{n} b_j t_j}{\sum_{j=1}^{n} b_j t_j} \tag{13}$$

Let $x_i = \frac{p_i}{p_n}, y = \frac{w}{p_n}, u_i = \frac{\pi_i}{p_n}$, and Equations (10) and (11) can be expressed as follows:

$$x_i - \sum_{j=1}^{n} a_{ij} x_j - \tau_i y = u_i \tag{14}$$

$$-\sum_{j=1}^{n} b_j x_j + y = 0 \tag{15}$$

We solve for x_s according to Cramer's Rule and derive [6] the following equation:

$$x_s = \sum_{i=1}^{n} \frac{(-1)^{i+s}\Delta_{is}}{\Delta} u_i \ (s = 1, \ldots, n-1) \tag{16}$$

$$1 = \sum_{i=1}^{n} \frac{(-1)^{i+n}\Delta_{in}}{\Delta} u_i \tag{17}$$

The relative price of the product produced by sector s will take the maximum value if sector s obtains the profits of all sectors, namely, the profits of all other sectors are zero, i.e. $ui = 0$ ($i \neq s$) when xs takes the maximum value; conversely, $ui = 0$ ($i \neq n$) when xs takes the minimum value, therefore the following equation is proposed:

$$x_{smax} = \sum_{i=1}^{n} (-1)^{2s} \frac{\Delta_{ss}}{\Delta} u_s \tag{18}$$

$$1 = (-1)^{n+s} \frac{\Delta_{sn}}{\Delta} u_s \tag{19}$$

$$x_{smax} = (-1)^{n+s} \frac{\Delta_{ss}}{\Delta_{sn}} \tag{20}$$

Likewise,

$$x_{smin} = (-1)^{n+s} \frac{\Delta_{ns}}{\Delta_{nn}} \tag{21}$$

Then, we prove the relationship between the relative prices of commodities and the labor theory of value:

When $n + s = 2m$ (m is a natural number),

$$x_{smax} = (-1)^{n+s} \frac{\Delta_{ss}}{\Delta_{sn}} = \frac{\Delta_{ss}}{\Delta_{sn}} = \frac{t_s}{t_n} \left(\frac{t_n}{t_s} \cdot \frac{\Delta_{ss}}{\Delta_{sn}} \right) = \frac{t_s}{t_n} \left(1 + \frac{t_n \Delta_{ss} - t_s \Delta_{sn}}{t_s \Delta_{sn}} \right) \tag{22}$$

$$t_n \Delta_{ss} - t_s \Delta_{sn} = \begin{vmatrix} 1 - a_{11} & \cdots & -a_{1s-1} & -a_{1s+1} & \cdots & -a_{1n}t_n - a_{1s}t_s & -\tau_1 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ -a_{s-11} & \cdots & 1 - a_{s-1s-1} & -a_{s-1s+1} & \cdots & -a_{s-1n}t_n - a_{s-1s}t_s & -\tau_{s-1} \\ -a_{s+11} & \cdots & -a_{s+1s-1} & 1 - a_{s+1s+1} & \cdots & -a_{s+1n}t_n - a_{s+1s}t_s & -\tau_{s+1} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ -a_{n1} & \cdots & -a_{ns-1} & -a_{ns+1} & \cdots & (1 - a_{nn})t_n - a_{ns}t_s & -\tau_n \\ -b_1 & \cdots & -b_{s-1} & -b_{s+1} & \cdots & -b_n t_n - b_s t_S & 1 \end{vmatrix} \tag{23}$$

Multiply column i of the determinant in Equation (23) [7] by t_i ($i = 1, \ldots, s-1$) and add it to column n, and then multiply column j by t_{j+1} ($j = s, \ldots, n-1$) and add it to column n, we obtain Equation (24):

$$t_n\Delta_{ss}-t_s\Delta_{sn}=\left(1-\sum_{j=1}^{n}b_jt_j\right)\begin{vmatrix}1-a_{11} & \cdots & -a_{1s-1} & -a_{1s+1} & \cdots & -a_{1n}t_n-a_{1s}t_s\\ \vdots & & \vdots & \vdots & & \vdots\\ -a_{s-11} & \cdots & 1-a_{s-1s-1} & -a_{s-1s+1} & \cdots & -a_{s-1n}t_n-a_{s-1s}t_s\\ -a_{s+11} & \cdots & -a_{s+1s-1} & 1-a_{s+1s+1} & \cdots & -a_{s+1n}t_n-a_{s+1s}t_s\\ \vdots & & \vdots & \vdots & & \vdots\\ -a_{n1} & \cdots & -a_{ns-1} & -a_{ns+1} & \cdots & (1-a_{nn})t_n-a_{ns}t_s\end{vmatrix}$$

$$=\left(1-\sum_{j=1}^{n}b_jt_j\right)\begin{vmatrix}1-a_{11} & \cdots & -a_{1s-1} & -a_{1s+1} & \cdots & -a_{1n-1} & \tau_1\\ \vdots & & \vdots & \vdots & & \vdots & \vdots\\ -a_{s-11} & \cdots & 1-a_{s-1s-1} & -a_{s-1s+1} & \cdots & -a_{s-1n-1} & \tau_{s-1}\\ -a_{s+11} & \cdots & -a_{s+1s-1} & 1-a_{s+1s+1} & \cdots & -a_{s+1n-1} & \tau_{s+1}\\ \vdots & & \vdots & \vdots & & \vdots & \vdots\\ -a_{n1} & \cdots & -a_{ns-1} & -a_{ns+1} & \cdots & -a_{nn-1} & \tau_n\end{vmatrix} \tag{24}$$

In the second determinant of Equation (24), multiply column i by $-t_i^*$ [8] ($i=1,\ldots,s-1$) and add it to column n, then multiply column j by $-t_j^*$ (j = s, . . ., n−2) and add it to column $n-1$, we obtain Equation (25):

$$t_n\Delta_{ss}-t_s\Delta_{sn}=\left(1-\sum_{j=1}^{n}b_jt_j\right)t_n^*\begin{vmatrix}1-a_{11} & \cdots & -a_{1s-1} & -a_{1s+1} & \cdots & -a_{1n-1}\\ \vdots & & \vdots & \vdots & & \vdots\\ -a_{s-11} & \cdots & 1-a_{s-1s-1} & -a_{s-1s+1} & \cdots & -a_{s-1n-1}\\ -a_{s+11} & \cdots & -a_{s+1s-1} & 1-a_{s+1s+1} & \cdots & -a_{s+1n-1}\\ \vdots & & \vdots & \vdots & & \vdots\\ -a_{n-11} & \cdots & -a_{n-1s-1} & -a_{n-1s+1} & \cdots & 1-a_{n-1n-1}\end{vmatrix} \tag{25}$$

$$=\left(1-\sum_{i=1}^{n}b_jt_j\right)t_n^*\Delta_{ss,nn,n+1n+1}=\frac{e}{e+1}t_n^*\Delta_{ss,nn,n+1n+1}$$

Therefore, the maximum value of the relative price is as follows:

$$x_{smax}=\frac{t_s}{t_n}\left(1+\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}\right) \tag{26}$$

When $n+s=2m+1$ (m is a natural number), the same is true for the following equation:

$$x_{smax}=\frac{t_s}{t_n}\left(1-\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}\right) \tag{27}$$

To sum up:

$$x_{smax} = \frac{t_s}{t_n}\left(1 + (-1)^{n+s}\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}\right) \tag{28}$$

The relationship between the minimum relative price and the labor theory of value can be obtained by a similar method:

$$x_{smin} = \frac{t_s}{t_n}\left(1 - \frac{e}{e+1}\cdot\frac{t_s^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{nn}}\right) \tag{29}$$

So, under the constraint of positive profit, the range of the relative price of a commodity is as follows:

$$\frac{t_s}{t_n}\left(1 - \frac{e}{e+1}\cdot\frac{t_s^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{nn}}\right) < \frac{p_s}{p_n} < \frac{t_s}{t_n}\left(1 + (-1)^{n+s}\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}\right) \tag{30}$$

It can be seen that under the constraint of positive profit conditions, the range of relative price is determined by the relative value, the rate of surplus-value and the technical structure of production inputs [9]. Different from the traditional general model of transforming value into production price, the value-price disequilibrium model proposed herein does not pre-assume the existence of an equilibrium general rate of profit; instead, it interprets the profit rate and market price as outcomes of disequilibrium dynamics. As mentioned earlier, the equilibrium profit rate is a general law obtained from theoretical derivation; however, the uncertainty of the economic system makes it almost impossible to reach equilibrium in reality and the direct equivalence of the ideal state on theoretical derivation to the economic reality leads to the discrepancy between the theoretical and empirical results. Therefore, the model of transforming value into production price based on the equilibrium perspective is more suitable for theoretical logic studies, whereas the value-price model of disequilibrium proposed in this paper uses the labor theory of value as a tool for grasping the uncertainty of market economic activities and can examine the transformation of market value into market price empirically. If we can prove in empirical data that the model's conclusion, i.e. Formula (30) holds, then it is strong proof of the validity of the labor theory of value in a more general sense. This validity is not only reflected in the fact that value is theoretically the basis for the price of production but also in the fact that the value of a commodity determines the range of market price, even under disequilibrium price conditions with non-average profit rates. In the next empirical section, we apply Formula (30) to the Chinese economic data to confirm the practical explanatory power of the labor theory of value.

4. Empirical analysis based on China's input–output table data

4.1 Data source and notes on the model's applicability

In this section, we empirically test the conclusion of the theoretical model constructed in Section 3 of this paper, i.e. Formula (30), using data from China's six sectoral input–output tables for 42 sectors published by the National Bureau of Statistics (NBS) for the years 2007–2020. The reason for choosing the 2007–2020 input–output table data is that, due to the changes in the standards for compiling input–output table data before and after 2007, it is not possible to compare the empirical data before and after 2007 directly. In addition, as the definition of unproductive sectors is still debated in academia, we exclude the finance, real estate, leasing, public administration and social organization sectors as unproductive sectors based on the common methods for treatment in existing studies.

However, the conclusion of Formula (30) is obtained from the physical input–output tables, revealing the range of the relative price of the unit commodity, whereas it is not possible to

specify a commodity's measurement unit in a value-based input–output table. Therefore, drawing on the methodology of Ochoa (1989), we calculate the relative price of commodity realized per unit of labor time in different sectors based on the value-based input–output table and transform the conclusion of Formula (30) into Formula (31):

$$1-\frac{e}{e+1}\cdot\frac{t_s^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{nn}}<\frac{\frac{p_s}{t_s}}{\frac{p_n}{t_n}}<1+(-1)^{n+s}\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}} \tag{31}$$

where $\frac{\frac{p_s}{t_s}}{\frac{p_n}{t_n}}$ represents the relative market price realized per unit of labor time in different sectors, which can be calculated based on the value-based input–output table.

Regarding the rate of surplus-value, the previous model defines the rate of surplus-value in the physical input–output table, $e=\frac{1-\sum_{j=1}^{n}b_jt_j}{\sum_{j=1}^{n}b_jt_j}$, which needs to be transformed into a form that can be calculated in the value-based input–output table. We estimate the in-kind wage vector according to the shares of products produced by various sectors in residential consumption, which is available in the value-based input–output table:

$$e=\frac{\frac{1}{w}-\sum_{j=1}^{n}\frac{b_jp_j}{w}\frac{t_j}{p_j}}{\sum_{j=1}^{n}\frac{b_jp_j}{w}\frac{t_j}{p_j}}=\frac{1-\sum_{j=1}^{n}b_jt_j}{\sum_{j=1}^{n}b_jt_j} \tag{32}$$

The left side of Equation (32) represents the rate of surplus-value in the value-based input-output table, and the right side represents the rate of surplus-value in the physical input–output table, numerically consistent with the left side. Thus, the rate of surplus-value can be estimated in the value-based input–output table corresponding to the physical input–output table from Equation (32). In addition, it is easy to transform the Δ from the physical input–output table to the value-based input–output table as follows:

$$\delta=\begin{vmatrix}(1-a_{11})\frac{p_1}{p_1} & \cdots & -a_{1n}\frac{p_n}{p_1} & -\tau_1\frac{1}{p_1}\\ \vdots & & \vdots & \vdots\\ -a_{n1}\frac{p_1}{p_n} & \cdots & (1-a_{nn})\frac{p_n}{p_n} & -\tau_n\frac{1}{p_n}\\ -b_1\frac{p_1}{w} & \cdots & -b_n\frac{p_n}{w} & \frac{1}{w}\end{vmatrix} \tag{33}$$

It can be obtained that in the value-based input–output table the following equations are proposed:

$$\frac{t_s^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{nn}}=\frac{\delta_{ss,nn,n+1n+1}t_S{}^*/p_s}{\delta_{nn}t_s/p_s} \tag{34}$$

$$\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}=\frac{\delta_{ss,nn,n+1n+1}t_n^*/p_n}{\delta_{sn}(p_n/p_s)t_s/p_s} \tag{35}$$

Equation (34) can be directly calculated from the physical input–output table, while Equation (35) cannot be directly calculated in the value-based input–output table due to the presence of p_n/p_s. In order to obtain the corresponding amount in the value-based input-output table, we transform $1+(-1)^{n+s}\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}$ into [10] the following equation:

$$1+(-1)^{n+s}\frac{e}{e+1}\cdot\frac{t_n^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{sn}}=\frac{(e+1)t_n\Delta_{ss}}{(e+1)t_n\Delta_{ss}-et_n^*\Delta_{ss,nn,n+1n+1}} \tag{36}$$

Based on the result of Equation (35), it is obvious that Equation (37) can also be calculated from the value-based input–output table.

At this point, all variables in Formula (38) can be calculated from the value-based input–output table. In the empirical test, we only need to test whether Formula (38) holds in the value-based input–output table so as to prove whether the range of price ultimately achieved by a commodity in market exchange is determined by the labor theory of value as well as its related abstract categories.

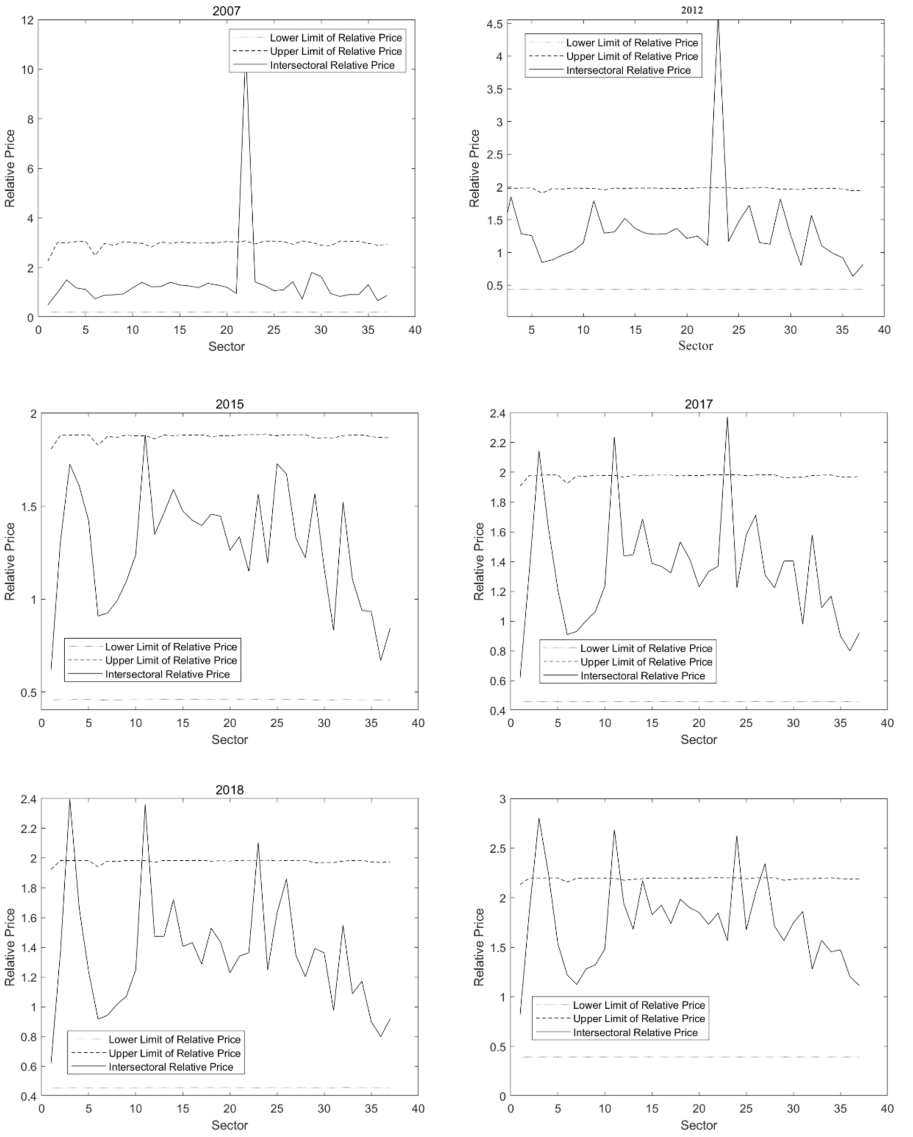
$$1-\frac{e}{e+1}\cdot\frac{t_s^*\Delta_{ss,nn,n+1n+1}}{t_s\Delta_{nn}}<\frac{\frac{p_s}{t_s}}{\frac{p_n}{t_n}}<\frac{(e+1)t_n\Delta_{ss}}{(e+1)t_n\Delta_{ss}-et_n^*\Delta_{ss,nn,n+1n+1}} \tag{37}$$

4.2 Analysis of empirical findings

(1) Empirical finding 1: The formation of relative market prices is based on the relative value of commodities

Marx's labor theory of value argues that, based on the claims of different sectors to a general rate of profit, the value of commodities is transformed into the price of production under the law of competition and that the form in which the law of value functions changes from market prices fluctuating around value to market prices fluctuating around the price of production. Although it is empirically difficult to correctly measure the equilibrium production price due to the complexity and volatility of the economic system, the basis for the formation of the production price is still the value of commodities. The research in this paper shows that the market price realized for commodities is the redistribution of the aggregate value of social production in different sectors and that the relative value of commodities, the rate of surplus-value and the technical structure of the inputs to production jointly determine the range of the relative price of commodities. The fluctuations in commodity prices can be explained by the labor theory of value. Figure 1 presents the estimation results based on China's input–output table data for 2007–2020. The results in Figure 1 show that the relative prices of each sector generally fall within the range of relative price set by the model in the study years, which validates the model and reveals the empirical validity of the labor theory of value in the sense of disequilibrium prices.

In the mainstream empirical testing methods, the fluctuation of prices, the selection of measurement indexes and the choice of production price solution will make the test results change and it is difficult to compare different studies horizontally, which also leads to the validity of the test being questioned. Compared with the mainstream empirical research methods for testing the labor theory of value, this paper abandons the idea of solving for the production price and then comparing the relationship between price, production price and value and turns to verify the relationship between fluctuations of the market price of commodities and their relative value under a looser theoretical constraint. The test method adopted in this paper is equivalent to expanding the existing method of "point estimation" to "interval estimation", verifying the practical explanatory power of the labor theory of value in a more general sense and providing a new approach for testing the labor theory of value. This



Note(s): The horizontal axis in Figure 1 indicates the sectors after excluding the non-productive sectors defined in this paper, and the order of sectors is consistent with that in the input-output table data published by the National Bureau of Statistics, which is shown in the Appendix of this paper (see Appendix A.2). The same applies to Figure 2 below

Source(s): Authors' own work

Figure 1. Sectoral relative prices and their ranges from 2007 to 2020 (excluding non-productive sectors)

complements the mainstream approach to testing the labor theory of value, which is to explain the role of value in determining market prices from the perspective of the range of market prices, in addition to the correlation and deviation indexes.

It should be noted that the empirical results presented in Figure 1 show relative prices in specific sectors exceeding the theoretical ceiling for the following reasons: Firstly, the model for calculating the theoretical upper limits on relative prices is based on the cost of production formed by the prices of each sector determined by the current period's technical coefficients. However, the market prices of the sectors counted in the input–output tables are affected by historical costs. Market prices are constantly fluctuating, and a model based on the current-period technology matrix would leave out the effect of historical costs. Secondly, the theoretical model is a "closed economy", and if there are differences between the prices of imported inputs and domestic prices, this will also have an impact on the results. Finally, input–output tables, as a result of statistical work, are subject to a certain amount of statistical error. This is particularly common in the waste materials recycling and processing sector where production statistics are not standardized, as the input-output coefficients are not standardized and are likely to cause the labor inputs involved and, thus, the aggregate surplus value to be underestimated. This leads to an underestimation of the upper limit of the production price, so that the market price, which contains the true amount of surplus value, is higher than the theoretical upper limit. The "exceptions" of market prices in specific sectors are unavoidable errors in the projection of theory into reality and do not affect the validity of the conclusion of this paper.

(2) Empirical finding 2: The range of relative market prices and the trend

The results in Figure 1 also show the magnitude and trend of the range of market prices in different years, which further explains the empirical validity of the labor theory of value. As can be seen from Figure 1, the limits of relative price available in various sectors are approximately at the same level in the years selected for this study. This is the result of the generalized linkage of production processes across sectors under modern technical conditions. In the *n*-sector input–output model, production in one sector often involves intermediate inputs from almost all other sectors, i.e. production is universally linked and the economic production process is a "circular flow". Therefore, the range of possible values of relative prices in any sector is determined to a large extent by all other sectors rather than by the sector itself. The fact that the range of relative prices in the different sectors is generally stable in the same range suggests that this range is governed by a universally operating economic law within the whole economic system. This universally operating law is the relevant abstract category of the labor theory of value set up in the model of this paper. This relationship is also reflected in Shaikh's (1984) study, which demonstrates that the deviations of the relative production prices of different commodities from their relative values are too minimal to consider. Since this deviation depends on the "vertically integrated" capital-to-labor ratio, the "vertically integrated" capital-to-labor ratios are approximately equal, while the direct investment capital-to-output ratios differ markedly between sectors. This is due to the fact that the result of vertical integration approximates a weighted average of the capital-output ratios of all sectors. Similarly, the convergence of relative price ranges across sectors is empirical evidence of the role of the labor theory of value.

In addition, the empirical results also reveal the trend of changes in the relative price range. The upper limit of the sectoral relative prices of the sector was around 2.5 in 2007 across sectors and has been basically maintained at around 2 since 2012; similarly, the lower limit of the sectoral relative prices has risen from around 0.2 in 2007 to approximately 0.5 across sectors. This shows that since entering the new era, China's market-oriented reform has effectively promoted the improvement of the socialist market economic system, and the law of value has played a role in resource allocation in a wider scope and at a deeper level, effectively regulating social production. As a result, the range of relative market prices that can be realized

in different sectors becomes smaller, the potential space for excess profits is further reduced and the law of competition plays a better role in the allocation of resources in the operation of the economy.

Finally, we also complement the validity of the empirical results by presenting the results when non-productive sectors are included. Figure 2 shows the results when non-productive sectors are included, and the results show that although the relative prices between sectors were still within the fluctuation range specified in the model in general, and the labor theory of value is still valid in this sense, the volatility of the market prices is much higher and there are more outliers out of the volatility range than the results in Figure 1. This suggests that the treatment of the non-productive sector in this paper is appropriate and supports the validity of the empirical results.

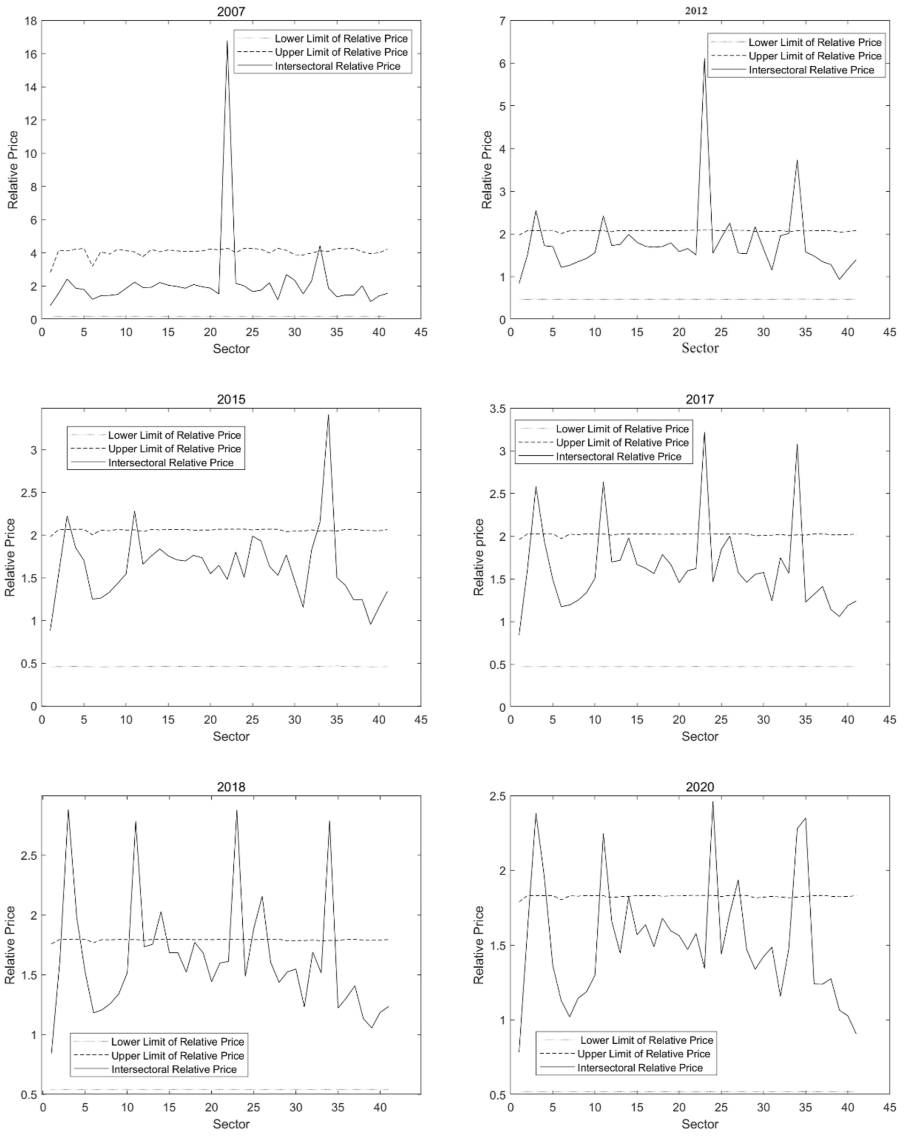
(3) Empirical finding 3: Relative price differences reflect inflows and outflows of value between industries

The empirical results of this paper can not only verify the validity of the labor theory of value but also provide a perspective for analyzing the inter-industry relationship from the labor theory of value, thereby providing suggestions on policymaking for promoting the balanced development of inter-industry and optimizing the layout of the industrial chain. Table 1 shows the five sectors with the highest and five sectors with the lowest relative market prices across all sectors from 2007 to 2020. A higher relative market price means that the surplus value obtained by the sector exceeds the average market level, which is conducive to the accumulation and development of the sector and vice versa.

First of all, Table 1 shows that in China's economic development, the sectors with high relative market prices per unit of labor time are mainly concentrated in natural monopoly industries such as oil, natural gas, electricity and mining, which, to an extent, reflects that these monopoly industries have become the main value-inflow sectors by virtue of their monopoly position in the market and obtained a relatively large share of the excess profits in the total social products. These natural monopoly industries are mainly composed of large state-owned enterprises (SOEs), which means that, on the one hand, it is necessary to continue to promote the market-oriented reform of SOEs to build a fair and competitive socialist market structure; on the other hand, the government can collect part of the surplus value by way of the profits paid by SOEs and use it to serve the enhancement of social welfare, for example, to increase the investment in some basic and leading technological fields, so that their research results can drive the high-quality development of related industries.

Secondly, sectors with low relative market prices mainly include traditional industries and public goods sectors, including agriculture, forestry, animal husbandry, fishery, textiles and services. Such sectors have seen an outflow of surplus value under market economic conditions, which is not conducive to the accumulation and development of the sector, and, therefore, require reasonable guidance and precise assistance based on the needs of employment and people's livelihood. Of course, the most fundamental development path is the transformation and upgrading of traditional industries through new technologies to improve production efficiency and achieve high-quality development.

Thirdly, some sectors that are in line with the law of industrial-technological upgrading and the development plan of the national economy, such as special purpose equipment, instrumentation, information transmission, computer services, software and information technology services, have also become sectors with relatively high relative market prices in half of the years in the examination period. This indicates, on the one hand, that the socialist market economy system with Chinese characteristics can strongly promote technological innovation and industrial upgrading and realize the progress of productive forces; on the other hand, the development of emerging industries is not stable and affected by the epidemic, they did not appear in the top five sectors with high relative market prices in the data of 2021. It can be seen that the sustained and rapid development of high-tech industries cannot be separated



Note(s): The horizontal axis in Figure 2 indicates the sectors after excluding the non-productive sectors defined in this paper, and the order of sectors is consistent with that in the input-output table data published by the National Bureau of Statistics, which is shown in the Appendix of this paper (see Appendix A.2)

Source(s): Authors' own work

Figure 2. Sectoral relative prices and their ranges from 2007 to 2020 (including non-productive sectors)

from the relevant policy support. Meanwhile, we should also be alert to the risks brought by new technologies and new forms of business, regulate and guide the sound development of capital, and prevent the disorderly expansion of capital to enable private enterprises to better

Table 1. The five sectors with the highest and lowest relative prices

Year	The five sectors with the highest relative market prices	The five sectors with the lowest relative market prices
2007	Waste fuel	Research and experimental development
	Information transmission, computer services and software	Food manufacturing and tobacco processing
	Wholesale and retail trade	Postal sector
	Oil and gas extraction	Health, social security and social welfare
	Transportation and warehousing	Farming, forestry, animal husbandry, fishery and services
2012	Waste fuel	Food and tobacco
	Oil and gas extraction	Health and social work
	Wholesale and retail trade	Accommodation and catering
	Petroleum, coking and nuclear fuel processing	Education
	Gas production and supply	Farming, forestry, animal husbandry, fishery and services
2015	Petroleum, coking and nuclear fuel processing	Food and tobacco
	Electricity and heat production and supply	Health and social work
	Oil and gas extraction	Accommodation and catering
	Gas production and supply	Education
	Mining and processing of metal ores	Farming, forestry, animal husbandry, fishery and services
2017	Recycling and processing of waste resources and waste materials	Health and social work
	Petroleum, coking and nuclear fuel processing	Food and tobacco
	Oil and gas extraction	Resident services, repairs and other services
	Gas production and supply	Education
	Metal smelting and rolling	Farming, forestry, animal husbandry, fishery and services
2018	Special purpose equipment	Mining and processing of non-metallic and other mineral ores
	Accommodation and catering	Textiles
	Manufacture of paper, printing and articles for culture, education and sport activity	Garment and apparel, footwear, headgear, leather, down and related products
	Instrumentation	Electrical machinery and equipment
	Information transmission, software and information technology services	Electricity and heat production and supply
2020	Oil and gas extraction sector	Food and tobacco
	Petroleum, coking and nuclear fuel processing	Education
	Repair of metal products, machinery and equipment	Textiles
	Production and supply of water	Health and social work
	Mining and processing of metal ores	Farming, forestry, animal husbandry, fishery and services

Source(s): Authors' own work

serve the construction of socialism without over-accumulation and distortion of sectorial structure.

Overall, the competitive relationship between sectors leads to the inflow and outflow of value across industries. It is essential to acknowledge the pivotal role of market competition in enabling capital mobility and value distribution when devising macroeconomic policies. Such policies should strive to establish a unified, open and orderly national market and, through the mechanisms of market competition, give full play to the law of value across various sectors, thereby promoting coordinated and sustainable development across industries. Moreover, targeted industrial policies must be implemented for specific sectors. Regulation of natural monopoly sectors should be strengthened, and market-oriented reforms for state-owned enterprises should be promoted. Besides, it is necessary to introduce corresponding industrial

policies to support some weak industries. For example, policy support should be provided to weak industries such as chip manufacturing, in addition to guiding government capital to participate in these high-tech industries. Additionally, for emerging fields such as the digital economy, it is necessary to speed up policymaking, not only to protect the income of specific factor owners such as owners of data and promote the sound development of the industry but also to regulate and guide the development of capital, preventing the disorderly expansion of capital.

The flow of value between sectors, reflected by relative price differentials, provides a reference for formulating industrial policies, which is a practical application of the analytical framework discussed in this paper. Future research should further quantify the level of industrial competition as indicated by relative prices, wherein the range of relative prices partly serves as a proxy for production prices as a reference standard. In the presence of production prices, the deviation of market prices from these production prices reflects the extent of market competition distortions. Production prices are the center of gravity for price fluctuation, functioning as a benchmark. The research in this paper indicates that it is difficult to directly determine the production price empirically; thus, their role as a reference is largely theoretical. In empirical research, the feasible range of relative prices can partially substitute for production prices, serving as a reference. The smaller the deviation between relative prices and the upper limit of relative price, the stronger the competitive position of that sector. This deviation value can act as a proxy variable for competition degree. Future empirical studies may incorporate more specific economic factors such as technical differences, capital turnover and the organic composition of capital for further modeling to analyze more specific macroeconomic problems.

Therefore, the theoretical framework presented in this paper effectively analyzes the flow of total social production value across various sectors, which enables an exploration of the competitive relationships between industries and informs adjustments to industrial policies aimed at promoting balanced development and optimizing the industrial chain layout. This not only broadens the application scope of the labor theory of value but also provides new insights for guiding economic practice through this theory. Further research can delve deeper into the mechanisms and theoretical logic behind higher or lower market prices in different sectors, incorporating more specific factors such as monopoly power, differences in technical structure and capital turnover, thereby offering robust theoretical support for fostering coordinated development among industries.

In summary, our empirical results validate the practical explanatory power of the labor theory of value across multiple dimensions, robustly complementing existing research. Furthermore, the theoretical framework presented herein can be utilized to analyze the flow of value between different sectors, presenting significant real-world implications. Of course, there are certain limitations, as evidenced by the results and ample room remains for further exploration. Firstly, the handling of non-productive sectors directly impacts our empirical findings, and there is still ongoing debate regarding the definition of these sectors. Although non-productive sectors do not participate in value production, they engage in value distribution, suggesting that simply excluding them might not be the optimal approach and warrants further examination. The empirical results obtained without excluding the non-productive sectors show more outliers in relative prices compared to the results with these sectors excluded, indicating that our exclusion approach is relatively reasonable. Secondly, the treatment of in-kind wages is somewhat rudimentary, raising questions about the accuracy of using the proportion of household consumption in different sectors as the share of in-kind wages, which requires additional discussion. As previously noted, the academic community also debates the theoretical assumption of an exogenously given vector of in-kind wage.

5. Conclusion

Since the abstract elements involved in the labor theory of value are challenging to observe directly, traditional research on the labor theory of value primarily focuses on theoretically

explaining its logical consistency and using it as an analytical tool for issues such as exploitation. The input–output method in political economy shifts the research on the labor theory of value towards empirical analyses, verifying the real-world explanatory power of the labor theory of value and broadening its research scope to include real economic problems. This paper summarizes and analyzes the empirical research method of the labor theory of value developed by scholars, including Shaikh and Ochoa, and points out that their limitations lie in the contentious solutions for the production price vector. Not only is there no consensus on the method of calculating production prices, but considering the complexity and volatility of economic movements, production prices can hardly be accurately calculated. Therefore, this paper abandons the approach of solving for the production price and, from the perspective of disequilibrium prices, derives a range of values for the relative market prices that can be realized for commodities based on more generous assumptions in an *n*-sector model. The model's conclusions indicate that the range of relative prices is jointly determined by relative values, the rate of surplus-value and the technical structure of production inputs. In other words, the range of relative prices is determined by elements related to the labor theory of value, reflecting the decisive role of value in determining market prices. This paper empirically validates the theoretical model's conclusions using data from China's input–output tables, supporting the scientific validity of the labor theory of value. The analysis framework based on disequilibrium prices we propose herein reveals, in a more general theoretical sense, the decisive role of value in the range of market price, proving the effectiveness of the labor theory of value. It supplements existing empirical research methods for testing the labor theory of value, further validates previous conclusions and expands the methods for experimental research on the labor theory of value.

This paper seeks to transcend the conventional analytical framework of the labor theory of value, which is primarily focused on affirming the theory's scientific validity. Based on existing empirical research paradigms related to the labor theory of value, the paper introduces an analytical framework that examines how market competition, technical conditions and other factors influence industrial structure and distribution relations through the lens of this theory. By examining value redistribution across diverse sectors, we evaluate the rationality of profit distribution within these sectors and provide a theoretical framework for advancing a high-level market economy and fostering high-quality industrial development from the perspective of Marxist political economy.

The analysis framework based on disequilibrium prices is constructed on the relatively flexible theoretical assumption that all sectors can achieve positive profits. The aim is to validate the practical explanatory power of the labor theory of value in a more generalized sense. In future research, we can apply this method to analyze more practical economic issues by introducing more specific constraints. For instance, we can consider incorporating factors such as the degree of market competition, the turnover rate of capital and the organic composition of capital into the disequilibrium-price model to analyze the structural differences in market prices and their ranges across sectors and estimate the inflows and outflows of value among these industries. By introducing concrete economic factors, this analytical framework can reveal the intrinsic connections between sectors from value production and realization, thereby providing corresponding policy recommendations to optimize the allocation of labor resources across industries, promote the construction of a modern industrial and economic system and facilitate high-quality economic development.

Notes

1. For a detailed discussion of the technical and theoretical limitations of existing empirical research methods, refer to the literature review in section 2 below.
2. In the Sraffa system on transformation, wages are paid post-factum, meaning that wages are not considered as advance capital. There have been studies examining the labor theory of value with both prepaid and postpaid wages. However, the conclusions from these studies do not differ significantly.

For the sake of brevity, this paper will only describe the solution of production prices based on the assumption of prepaid wages.

3. In this paper, we only introduce three solutions of the transformation problem to solve for the production price. As for other solutions of the transformation problem and the advantages and disadvantages of different solutions, they have little to do with the theme of this paper, so we will not discuss them here.

4. Strictly speaking, the production price vector does not necessarily need to be measured in monetary units, because, in theory, production prices result from the redistribution of surplus value based on value. However, production prices calculated based on value-based input–output tables and nominal wages are indeed measured in monetary units.

5. It should be noted that the assumption of positive profit in n-sector model is theoretically justified. In an *n*-sector economy, the economic system is turbulent and complex. While individual entities may experience operating losses, each sector is an aggregate of numerous firms. Thus, it is evidently reasonable to assume that, on average, any given sector achieves positive profits.

6. where

$$\Delta = \begin{vmatrix} 1-a_{11} & \dots & -a_{1n} & -\tau_1 \\ \vdots & & \vdots & \vdots \\ -a_{n1} & \dots & 1-a_{nn} & -\tau_n \\ -b_1 & \dots & -b_n & 1 \end{vmatrix}$$

and Δ_{nn} indicates the sub-formulas after row *n* and column *n* are removed, Δ_{sn} the sub-formulas after row *s* and column *n* are removed, $\Delta_{ss,nn,n+1n+1}$ the sub-formulas after row *s*, *n*, *n+1* and column *s*, *n*, *n+1* are removed.

7. Move the elements of Δ_{sn} in column *s* to column *n−1* and get:

$$\triangle_{sn} = (-1)^{n-s-1} \begin{vmatrix} 1-a_{11} & \cdots & -a_{1s-1} & -a_{1s+1} & \cdots & -a_{1s} & -\tau_1 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ -a_{s-11} & \cdots & 1-a_{s-1s-1} & -a_{s-1s+1} & \cdots & -a_{s-1s} & -\tau_{s-1} \\ -a_{s+11} & \cdots & -a_{s+1s-1} & 1-a_{s+1s+1} & \cdots & -a_{s+1s} & -\tau_{s+1} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ -a_{n1} & \cdots & -a_{ns-1} & -a_{ns+1} & \cdots & -a_{ns} & -\tau_n \\ -b_1 & \cdots & -b_{s-1} & -b_{s+1} & \cdots & -b_s & 1 \end{vmatrix}$$

Therefore, determinant combination of $t_n\Delta_{ss} - t_s\Delta_{sn}$ can be carried out.

8. We define it as follows: $t_i^* = \sum_{j\neq s,n}^{n-1} a_{ij}t_j^* + \tau_i \, (i = 1, \cdots, n)$

9. In Equation (27), Δ and t_i^* can be interpreted as indexes reflecting the characteristics of the technical structure of production inputs in the *n*-sector model, which is not intuitively represented in the *n*-sector disequilibrium model, but its meaning can be visualized in a two-sector model. In the two-sector model, the range of relative prices is as follows:

$$\left(1-\tfrac{e}{1+e}\right)\tfrac{t_1}{t_2} < \tfrac{p_1}{p_2} < \left(1+\tfrac{e}{k_2(1+e)}\right)\tfrac{t_1}{t_2}$$

where k_2 is the organic composition of the second sector, and the specific derivation process of the two-sector model is shown in Appendix A.1 to this paper. The above formula delineates that the range of relative prices between different production sectors under the constraint of positive profit conditions is determined by the labor value, the rate of surplus-value and the organic composition of capital. In an *n*-sector model, the organic composition that reflects the technical structure characteristics of production inputs is transformed into a more complex mathematical form as presented in the main text.

10. From Equations (24), (25) and (26), we have $t_S\Delta_{sn} = t_n\Delta_{ss} - (-1)^{n+s}\frac{e}{e+1}t_n^*\Delta_{ss,nn,n+1n+1}$, which can be transformed into Equation (34) by substituting into Equation (35).

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(The Appendix follows overleaf)

Appendix

A.1 The value-price disequilibrium model in a two-sector model

Consider the relationship between relative price and relative value of commodities in a two-sector model. Assuming that sector I is the sector of the means of production, commodity 1 capital goods, sector II the sector of the means of consumption, and commodity 2 consumer goods, Table 1 describes the economic production process in the two-sector model.

Table A1. Production process

	Input Commodity 1	Labor	Output Commodity 1	Commodity 2
Sector I	a_1	τ_1	1	0
Sector II	a_2	τ_2	0	1

Source(s): Authors' own work

The conditions under which capitalists in both sectors can earn positive profits are:

$$p_1 > a_1 p_1 + \tau_1 w \tag{A1}$$

$$p_2 > a_2 p_1 + \tau_2 w \tag{A2}$$

$$w = b p_2 \tag{A3}$$

where w represents the in-kind wages of workers, t_1 and t_2 the labor value of unit commodity 1 and commodity 2, respectively. The labor value of commodity can be expressed as follows:

$$t_1 = a_1 t_1 + \tau_1 \tag{A4}$$

$$t_2 = a_2 t_1 + \tau_2 \tag{A5}$$

The rate of surplus-value e and the organic composition of capital of sector II, k_2, are:

$$e = \frac{1 - b t_2}{b t_2} \tag{A7}$$

$$k_2 = \frac{\tau_2}{a_2 t_1} \tag{A8}$$

Solving the system of joint equations, we can get the following relationship between relative prices and relative values of commodities:

$$\left(1 - \frac{e}{1+e}\right)\frac{t_1}{t_2} < \frac{p_1}{p_2} < \left(1 + \frac{e}{k_2(1+e)}\right)\frac{t_1}{t_2} \tag{A9}$$

A.2 Order of sectors in China's 42-sector input-output table

Table A2. Order of sectors in China's input-output table

Code	Sector	Code	Sector	Code	Sector
01	Farming, forestry, animal husbandry, fishery and services	15	Metal products	29	Wholesale and retail trade
02	Mining and processing of coal	16	General purpose equipment	30	Transportation, warehousing and postal services
03	Oil and gas extraction products	17	Special purpose equipment	31	Accommodation and catering
04	Mining and processing of metal ores	18	Transport equipment	32	Information transmission, software and information technology services
05	Mining and processing of non-metallic and other mineral ores	19	Electronic machinery and equipment	33	Finance
06	Food and tobacco	20	Communication equipment, computers and other electronic equipment	34	Real estate
07	Textiles	21	Instrumentation	35	Leasing and business services
08	Garment and apparel, footwear, headgear, leather, down and related products	22	Other manufactured products	36	Scientific research and technical services
09	Wood products and furniture	23	Recycling and processing of waste resources and waste materials	37	Management of water conservancy, environment and public facilities
10	Manufacture of paper, printing and articles for culture, education and sport activity	24	Repair of metal products, machinery and equipment	38	Resident services, repairs, and other services
11	Petroleum, coking, and nuclear fuel processing	25	Electricity and heat production and supply	39	Education
12	Chemicals	26	Gas production and supply	40	Health and social work
13	Non-metallic mineral products	27	Water production and supply	41	Culture, sports and recreation
14	Metal smelting and rolling	28	Architecture	42	Public administration, social security and social organization

Source(s): Authors' own work

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